# Common Core's Standards Still Don't Make the Grade 

Why Massachusetts and California Must Regain Control Over Their Academic Destinies

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A Pioneer Institute White Paper
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## Pioneer's Mission

Founded in 1988, Pioneer Institute is a non-partisan public policy think tank committed to keeping Massachusetts economically competitive and to strengthening the core values of an open society. To inspire market-driven policy reforms, Pioneer promotes individual freedom and responsibility and limited, accountable government. The Institute has changed the intellectual climate in Massachusetts by commissioning timely and rigorous academic studies from leading scholars. Pioneer injects new ideas into the public debate through forums and lectures, transcripts, the media, and outreach to legislators, business groups and the general public.

## Pioneer's Centers

This paper is a publication of Center for School Reform, which seeks to increase the education options available to parents and students, drive system-wide reform, and ensure accountability in public education. The Center's work builds on Pioneer's legacy as a recognized leader in the charter public school movement, and as a champion of greater academic rigor in Massachusetts' elementary and secondary schools. Current initiatives promote choice and competition, school-based management, and enhanced academic performance in public schools.

The Center for Economic Opportunity seeks to keep Massachusetts competitive by promoting a healthy business climate, transparent regulation, small business creation in urban areas and sound environmental and development policy. Current initiatives promote market reforms to increase the supply of affordable housing, reduce the cost of doing business, and revitalize urban areas.

The Shamie Center for Better Government seeks limited, accountable government by promoting competitive delivery of public services, elimination of unnecessary regulation, and a focus on core government functions. Current initiatives promote reform of how the state builds, manages, repairs and finances its transportation assets as well as public employee benefit reform.

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## Executive Summary

The case for national standards rests in part on the need to remedy the inconsistent and inferior quality of many state standards and tests in order to equalize academic expectations for all students. The argument also addresses the urgent need to increase academic achievement for all students. In mathematics and science in particular, the United States needs much higher levels of achievement than its students currently demonstrate for it to remain competitive in a global economy.

In the past five months, the Pioneer Institute and the Pacific Research Institute have sponsored three White Papers that analyzed the evolving drafts of the two types and sets of standards that Common Core was developing: "college and career readiness standards" for common tests to determine preparedness for college-level work in mathematics and reading, and K-12 grade-level standards for grade-level tests in mathematics and English language arts (ELA). Those Papers addressed different drafts and issues.

The purpose of this White Paper was to determine whether Common Core's final standards provide a stronger and more challenging framework for mathematics and ELA curricula than do California's current standards and Massachusetts' current (2001) and revised draft (2010) standards. In other words, can Common Core rightfully claim that its standards will prepare more high school graduates for authentic college-level work than California's and Massachusetts' standards do?

Results of our review of four sets of English language arts standards: For our analysis of the four sets of ELA standards, we shortened and slightly revised the review form used in the 1997, 2000, and 2005 reviews of state English standards for the Thomas B. Fordham Institute. The items eliminated were, for the most part, not relevant for the comparisons we intended to make. Most of the 20 items retained for the 2010 review form are similar or identical in wording to their counterparts in the 2005 review form, as are the rubrics for the rating scale. Table 1 summarizes the results of this analysis

Results of our review of three sets of mathematics standards: The National Mathematics Advisory Panel (NMAP) proposed three clusters of concepts and skills for K-8: Fluency with Whole Numbers, Fluency with Fractions, and Particular Aspects of Geometry and Measurement. For this review, we compared how Common Core's mathematics standards address one or two key topics in each cluster with the way those topics are addressed by California's mathematics standards, by the 2010 draft standards in Massachusetts, and by standards used in high-achieving countries. We then compared how each of these documents addresses preparation for Algebra I in grade 8 and high school mathematics.

In K-8: All three sets of standards we reviewed develop fluency with the arithmetic of integers by grade 5 as recommended by NMAP, with the notable exception of division, which Common Core defers to grade 6. Although Common

Table 1: Average Points per Section and Total Average for Four ELA Documents

|  | CC | CA | MA | DMA |
| :---: | :---: | :---: | :---: | :---: |
| Reading Pedagogy and Independent Reading | 3.6 | 3.6 | 4 | 4 |
| Value of Literary Study | 3 | 2.3 | 4 | 4 |
| Organization and Disciplinary Coverage of the Standards | 2.4 | 3.5 | 3.7 | 3.8 |
| Quality of the Standards | 2.5 | 3.5 | 4 | 4 |
| Total | 2.7 | 3.4 | 3.9 | 3.9 |

Core has a well-developed learning progression for common fractions, it fails to build on money to introduce decimal fractions early, it neglects development of flexibility with fraction representations, it provides no material on teaching and using least common denominators beyond the simple multiplication of denominators, it fails to teach prime factorization at any grade level, and it seriously overuses visual fractions. Overall, Common Core's preparation for Algebra I falls a year or two behind the standards in California and high achieving nations.

In 9-12: There are also many weaknesses in Common Core's high school standards. Compared with the content of the standards in California and Massachusetts for Algebra I, Geometry, and Algebra II, the content of Common Core's standards for these three basic courses shows low academic expectations for its definition of "college readiness." Finally, Common Core's replaces the traditional Euclidean foundations of school geometry with an experimental approach to the study of middle and high school geometry that has not been widely used elsewhere in the world, or considered effective where it was tried out.

## Conclusions

1. Common Core's literature and reading standards in grades 9-12 do not prepare students for college and career better than those in California and Massachusetts. Common Core's high school standards fall well short of those in California and in Massachusetts 2001 and 2010 in specificity of literary and cultural content. By adopting Common Core's standards for their own, California and Massachusetts significantly weaken the intellectual demands on students in the areas of language and literature. They also weaken the base of literary and cultural knowledge needed for actual college-level work now implied by each state's current or draft standards.
2. English teachers cannot readily teach to all of Common Core's literature and reading standards in grades 9-12. Common Core expects English
teachers to spend over 50 percent of their time addressing literary nonfiction and informational texts such as seminal U.S. documents and U.S. Supreme Court decisions. Given what they are prepared to teach based on their undergraduate or graduate coursework, English teachers cannot teach to many of Common Core's informational reading standards and they are unlikely to try to do so.
3. English teachers will likely be held accountable for most results on high school reading tests. It is likely that English teachers will be held accountable for much more than 50 percent of the results on Reading tests based on Common Core's college and career readiness standards. A clear answer to this question is not possible because Common Core speaks with forked tongue on who will teach students to read informational texts. Common Core wants reading passages on high school college-readiness tests to reflect the National Assessment of Educational Progress's distribution of passages for its high school reading tests: 70 percent informational, 30 percent literary. It is highly unlikely that English teachers will be held accountable for only a little more than 30 percent of the Reading test score.
4. Common Core's standards make a coherent K-12 ELA curriculum unattainable. Unlike standards that point to the general cultural or literary knowledge (as well as the generic thinking and language skills) needed at each grade level, Common Core's "anchor" and gradelevel standards not only provide no intellectual base or structure for a curriculum, they actually prevent one from emerging.
5. Common Core's ELA standards will require drastic changes in academic, preparation, and professional development programs for prospective or current English teachers. English teachers will need to take a significant amount of academic coursework (or professional development) in history and political science to understand the historical context, philosophical influences, unique features, and national and
international significance, historically and today, of the seminal documents they are being required to teach students how to read. They will also have to undergo professional training in reading scientific and other discipline-based texts.
6. Although Common Core's standards represent a laudable effort to shape a national curriculum, the draft-writers chose to navigate an uncharted path and subject the entire country to a largescale experimental curriculum rather than build on the strengths that can be documented in Massachusetts or California. Consequently, by grade 8 their mathematics standards are a year or two behind the National Mathematics Advisory Panel's recommendations, leading states, and our international competitors. No media discussion took place after several experts on the Validation Committee refused to sign off on Common Core's standards, and the public has been left with the incorrect impression that English scholars, mathematicians, and high school English and mathematics teachers are unified in support of its ELA and mathematics standards. Common Core's mathematics standards miss chunks of content recommended by the National Mathematics Advisory Panel for K-8 and inexplicably leave large holes in mathematics content currently in the high school curriculum.

Our analysis of Common Core's mathematics and ELA standards, and the evidence we provide, do not support the conclusion drawn by many other reviewers that Common Core's standards provide a stronger and more challenging framework for the mathematics and English language arts curriculum than (or an equally as challenging framework as) California's and Massachusetts' standards have provided. Common Core's standards will not prepare more high school students for authentic college-level work than standards in these states have prepared. To the contrary, they may lead to fewer high school students prepared for authentic college-level work.

We offer these recommendations to states that are adopting Common Core's standards.

- Delete the label of "college and career readiness standards" or "college and career readiness anchor standards" on all of Common Core's standards.
- Exercise caution in developing textbooks and other curricular materials based on Common Core's ELA standards until several key standards are appropriately placed and/ or correctly written and several conceptual errors are corrected.
- Develop incentives to encourage high school sophomores or juniors deemed "college-ready" to complete grades 11 and 12 and take advanced mathematics and science courses that prepare them for authentic college-level coursework rather than leave high school to enroll in an open admissions institution.
- Ask Congress to require the National Assessment Governing Board to conduct a High School Transcript Survey (HSTS) every two years to monitor high school mathematics course-taking.


# Common Core's Standards Still Don't Make the Grade 

## I. Introduction

The case for national standards rests in part on the need to remedy the inconsistent purposes and inferior quality of many state standards and tests in order to equalize academic expectations for all students. The argument also addresses the urgent need to increase academic achievement for all students. In mathematics and science in particular, the United States needs much higher levels of achievement than its students currently demonstrate for it to remain competitive in a global economy. In 2009, with the encouragement of the U.S. Department of Education (USED) and the support of the Bill and Melinda Gates Foundation, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) agreed to sponsor the Common Core State Standards Initiative (CCSSI) to develop mathematics and English language arts standards (ELA) for "voluntary" adoption by all states. In turn, USED decided to award up to 70 (of 500) points to states committing themselves to adopt Common Core's standards and the common assessments to be based on them in their 2010 applications for Race to the Top (RttT) funds. USED also decided to tie states' receipt of Title I funds, in a re-authorization of the No Child Left Behind Act (NCLB), to high school tests of college readiness.

In the past five months, the Pioneer Institute and the Pacific Research Institute have sponsored three White Papers that analyzed the evolving drafts of the two types and sets of standards that Common Core was developing: "college and career readiness standards" for common tests to determine preparedness for college-level work in mathematics and reading, and K-12 grade-level standards for grade-level tests in mathematics and English language arts. ${ }^{1}$

- The White Paper issued in February 2010 pointed out deficiencies in Common Core's September 2009 draft of its college and career readiness standards and in its January 2010 draft of grade-level standards, which

Common Core describes as a grade-by-grade "translation" of its readiness standards.

- The White Paper issued in April 2010 examined Common Core's March "public comment" draft to determine how much progress had been made in addressing deficiencies in both subjects and in both types of standards and to point out areas needing further revision so that Common Core's standards could serve as the basis for valid and reliable tests of college-preparedness as well as of grade-level progress toward that goal.
- The White Paper issued in May 2010 highlighted the low academic level Common Core has set for its college readiness standards and the lack of evidence to support this low level. It also suggested what changes should be made to both sets and types of standards so that test developers can develop tests that make college-readiness mean readiness for actual college freshman coursework.

On June 2, 2010, Common Core released the final version of its standards. The purpose of this White Paper differs from the purpose of the first three White Papers. This Paper seeks to determine whether Common Core's final standards provide a stronger and more challenging framework for the mathematics and English language arts curriculum than do California's current standards and Massachusetts' current (2001) and revised draft (2010) standards so that more students are prepared for authentic college-level work when they graduate from high school. It also discusses the implications of Common Core's standards for curriculum and textbook development, teacher preparation, and professional development in each state.

## II. Why the Comparison with California and Massachusetts?

Although there are many reasons for comparing Common Core's standards with those in

California and Massachusetts, the central reason concerns the academic consequences of adopting its grade-level and "college and career readiness standards" for advanced mathematics coursetaking in each state's high schools. Both states are viewed as having more rigorous standards in both subjects than most other states. ${ }^{2}$ Many organizations and individuals have long considered and used California's mathematics standards as the "gold standard" by which to judge other states' mathematics standards. Similarly, many organizations and individuals have long considered and used the Bay State's ELA standards as the "gold standard" by which to judge other states' ELA standards. In addition, empirical evidence in both states attests to the effectiveness of their standards (see Why Race to the Middle? February 2010 White Paper), although Massachusetts has much more evidence than California does. Use of high school mathematics and ELA standards that may be only somewhat more challenging (at best) than those each state now uses as the basis for competency tests for a high school diploma would translate into a larger number of students leaving high school unprepared for actual college-level work than is now the case. Why would this be the case?

Common Core changes what passing a test based on high school standards will signify. Current high school tests in most if not all states are designed to determine competency for a high school diploma, not college-level work, and state tests and their pass scores are based on standards designed with that purpose in mind. On the other hand, passing common tests based on Common Core's high school standards will deem grade 10 or 11 students competent for college-level work, not just a high school diploma, even though they have not yet taken the work they would normally take in their junior and/or senior years of high school. ${ }^{3}$ Moreover, results of other tests also reflecting these high school standards will be used by the National Center on Education and the Economy (NCEE) to encourage lower-achieving students who pass them to forgo the last year or two of high school to enroll immediately in an open
admissions post-secondary degree program. ${ }^{4}$ These students are to be placed, or will expect placement, in credit-bearing-not remedialcourses since the tests they have passed have deemed them college-ready. Thus, standards that may be, at best, only somewhat more challenging than those now used for determining competency for a high school diploma will open a floodgate for college freshmen with even less knowledge and skill for actual college-level work than most of today's college freshmen, most of whom have completed grades 11 and 12 .
A. California's Concerns: We discuss the situation for California and Massachusetts separately because each state has somewhat different reasons for looking carefully at the farreaching academic policies Common Core has, without public discussion, built into its standards documents. As noted above, the first concern for California may well be about trading rigorous standards for standards that may or may not be equally rigorous but, either way, will serve as the basis for tests granting a higher academic status than "high school graduate" to those who pass them and enable grade 10 or 11 students to bypass grade 11 or 12 . So far, there is no evidence that this may be beneficial to lower-achieving students. While formal requirements for college admission in California are three years of mathematics (Algebra I, Geometry, and Algebra II), only 25 percent of the students who take Algebra II in their junior year are fully or conditionally ready for non-remedial mathematics in college as determined by the Early Assessment Program, which is given in the junior year. But 88 percent of the grade 11 students who took Algebra II in grade 10 and then took an additional year of mathematics in their junior year are fully or conditionally college-ready. ${ }^{5}$ These correlations suggest that it may be desirable for students to take Algebra I in grade 8 so that they can take two more years of mathematics courses after taking Algebra II in grade 10. These additional mathematics courses in grade 11 and/or 12 are important to take because they clearly contribute to college readiness.

California's second concern is whether Common Core's K-7 mathematics standards appear to be sound enough to prepare all students for an authentic Algebra I course in grade 8, a state policy. Empirical evidence suggesting the effectiveness of the state's K-7 and Algebra I mathematics standards appears in the form of regularly increasing percentages of students taking and passing Algebra I in grade 8 at higher levels of performance. ${ }^{6}$

A third concern for California is the fact that as a textbook adoption state it has already invested a great deal in funding school-selected textbooks recommended as compatible with its standards. New sets of standards will necessitate huge costs for new textbooks and professional development, points recently noted by the Virginia Board of Education in its explanation for not adopting Common Core's standards. ${ }^{\text {? }}$

The fourth concern would be normal in any state with a well-developed set of support structures that over time appear to have facilitated an increase in student achievement. Large changes in state standards introduce a period of instability and misaligned support structures, with a possibly negative impact on student achievement. In both California and Massachusetts, a long period of stability in the state's standards - a decade or more - has enabled policy makers to pay attention to the development of strong and reliable state assessments and to the strengthening of teacher training programs, licensure tests, and professional development. The strength of these support structures in Massachusetts is well known, but California has also made large strides in these areas in the context of a large shift in its demographics. In the past 17 years, the percentage of Hispanic students grew from about 30 percent to over 50 percent while the percentage of those classified as white dropped from about 50 percent to 28 percent. A sudden change in state standards will undermine those support structures at a time of economic recession and may jeopardize recent increases in student achievement.
B. Massachusetts'Concerns: For Massachusetts, the first concern likewise arises from trading rigorous standards for standards that may or may not be equally rigorous but, either way, will serve as the basis for determining readiness for college, not a high school diploma. Moreover, there is abundant empirical evidence suggesting the effectiveness of the state's current standards for all groups of students in both mathematics and ELA, and in science as well. ${ }^{8}$ The academic rigor of the state's annual assessments, which are based on the state's standards, has long been acknowledged as another major factor in increasing the academic achievement of the state's students. And its regulations for teacher preparation programs and licensure tests-also based on its standards-are considered among the most rigorous in the country. It is not clear that Common Core's standards could serve as an effective replacement for the standards now at the center of these systemically linked academic components since Common Core's standards have no track record of use anywhere, no research evidence to support them, and are not internationally benchmarked.

As in California, a second concern in Massachusetts is whether Common Core's K-7 mathematics standards appear to be sound enough to prepare all students for an authentic Algebra I course in grade 8. Although enrollment in an authentic Algebra I course in grade 8 is not state policy, many school districts mandate that all students take Algebra I in grade 8. More than 50 percent of the state's students enroll in Algebra I in or by grade 8, a percentage that has been increasing regularly in the past decade.

Although Massachusetts is not a textbook adoption state (i.e., schools choose-and pay for-whatever textbooks they wish), the textbooks and professional development a school pays for must be aligned with the state's standards for accountability if the school fails to achieve Adequate Yearly Progress (AYP) as part of NCLB. Because a large number of schools in Massachusetts have failed to achieve AYP, a
considerable amount of money has been invested in textbooks and teacher training in the state. New standards will require further investment in textbooks and professional development.

A fourth concern is a possible decline in advanced mathematics course-taking in high school by students in the broad middle third (or higher) of the state's high school-age population if Common Core's standards are adopted. If the common high school tests are no more (or only somewhat more) rigorous than the state's current grade 10 tests (to be referred to as MCAS), students who leave high school after grade 10 or 11 to enroll in a college degree program may be even less prepared for authentic college-level work than those who now complete local high school graduation requirements. That is because students who pass the new tests will be deemed ready for creditbearing freshman coursework in college even though they have not completed their last year or two of high school and taken more advanced mathematics and English coursework there.

A 2008 report by the state's Board of Higher Education (BHE) and Department of Education found that 50 percent of the students who had (1) passed the grade 10 mathematics MCAS test at the "Needs Improvement" level, (2) graduated from a public high school in 2005, and (3) enrolled in a public college in the fall of 2005 had been placed in a remedial mathematics course. ${ }^{9}$ This means that the pass score for the grade 10 mathematics MCAS does not indicate college readiness. Overall, the figures have not improved much since 2005. As the 2008 report noted, regardless of performance level, 37 percent of the state's public high school graduates in 2005 had been placed in at least one remedial course in their first semester in a public college, and the figure was 36 percent for the freshman class entering in fall $2008 .{ }^{10}$

One source of this problem is the relatively small number of students inthe state'spublichigh schools who take a mathematics course in their senior year, as implied by the prominent recommendation in

MassCore (a state-recommended high school course of study) that students take a mathematics course in their senior year, ${ }^{11}$ despite the high percentage ( 82 percent) who have already taken Algebra II according to a Department study. ${ }^{12}$ Only 29 percent of the state's school districts require four years of mathematics and the BHE does not require students who seek to enroll in a Massachusetts public post-secondary institution to take four years of mathematics in high school despite some data showing that students who take mathematics in their senior year are more likely to pass their first college mathematics course than those who don't. ${ }^{13}$ It is not surprising that many freshmen in public post-secondary institutions must enroll in a developmental (remedial) course; they may not have studied any mathematics at all for well over a year, taken a course beyond Algebra II, or done well in the Algebra II course they took. Nationally, only 61 percent of U.S. high school students take a mathematics course beyond Algebra II before graduation. ${ }^{14}$ Tests that allow grade 10 or 11 students who pass to be deemed college-ready and to enroll directly in a public post-secondary institution will do a disservice to the broad middle third of the state's high school-age population. They should be expected to take more advanced mathematics courses in their junior and senior years (e.g., Algebra II, trigonometry, or pre-calculus) before they matriculate at a state college.

A final concern is less directly obvious-the replacement of the current set of teacher licensure tests with easier ones. This is a long sought-after goal by those promoting a bill in the legislature to replace the state's current licensure tests with those sold by Educational Testing Service (i.e., PRAXIS tests). Once the state's standards are changed, there will be pressure to change and weaken the state's tests of subject matter knowledge, which assess prospective teachers on their conceptual understanding of the state's content standards in the field of their license. Instead of upgrading admissions policies, many teacher preparation programs would prefer to allow more academically underprepared students
into their programs based on an evidencefree belief that at risk children do not need academically competent teachers. That less than 60 percent of prospective elementary teachers pass the state's new elementary mathematics test and that less than 60 percent of prospective early childhood teachers pass their general curriculum test are facts that inexplicably seem to signal to teacher education faculty or to the Board of Higher Education an illogical inference--that the licensure tests now assuring parents that their children's teachers have adequate knowledge of their subjects need to change, rather than admissions policies to teacher preparation programs.

## III: Review of English Language Arts Standards

We analyzed four sets of ELA standards: California's 1997 ELA standards; Massachusetts's 2001 ELA standards, together with the 2004 supplement; Massachusetts's draft 2010 ELA standards; and Common Core's June 2010 ELA standards. California's standards have never been revised since they were approved in 1997. The 2004 supplement to the Massachusetts 2001 standards was developed to address NCLB's requirement for tests at every grade level from grade 3 to grade 8: the 2001 standards were designed for two-year grade spans, In 2008, Massachusetts again began the process of revising its ELA standards, as mandated by the Education Reform Act of 1993. But before the process was completed, Common Core's project had begun. A draft of the revised standards was sent to Common Core's draft writers in 2009 as a resource for their work, and for their final version they drew heavily on many standards in that 2009 draft (now dated 2010). However, Common Core's draft writers did not usually adhere to their original wording, gradelevel placement, or, in many cases, meaning. In fact, they so often distorted their meaning with poor paraphrasing or inappropriate examples that much of the borrowed material ended up in
standards that are not consistently interpretable or teachable. ${ }^{\text {. }}$

For the analysis of the four sets of ELA standards, we adapted the review form used in the 1997, 2000, and 2005 reviews of state English standards for the Fordham Institute. The review form used for the 2010 review is much shorter; the items eliminated were, for the most part, not relevant for the comparisons we intended to make. Most of the 20 items retained for the 2010 review form are similar or identical in wording to their counterparts in the 2005 review form, as are the rubrics for the rating scale. ${ }^{15} \mathrm{We}$ also retained the $0,1,3$, and 4 rating scale used in the 2005 review. (A 2 was to be used only when it was completely unclear what was in a document.)

The following legend indicates the source of the standards reviewed on the following pages.

[^0][^1]
## Review of Four English Language Arts Standards Documents

## A. Reading Pedagogy and Independent Reading

1. It expects explicit and systematic instruction in decoding skills in the primary grades as well as use of meaningful reading materials and an emphasis on comprehension.

## CC $\quad$ Rating: 3

Research in reading is clearly used to inform the acquisition of decoding skills. There is good coverage of key comprehension skills across subject areas, as well as use of meaningful reading materials. However, none of the objectives on phonics and word analysis skills in grades K-3 expects students to apply these skills both in context and independent of context to ensure mastery of decoding skills. Only in grades 4 and 5 are students expected to read accurately unfamiliar words "in context and out of context." This standard needs to be in the primary grades as well. Its placement at only grades 4 and 5 badly misinforms reading teachers in the primary grades.

## CA $\quad$ Rating: 3

Research in reading is clearly used to inform the acquisition of decoding skills. There is good coverage of key comprehension skills across subject areas, as well as use of meaningful reading materials. However, none of the objectives on phonics and word analysis skills in grades K-3 expects students to apply these skills both in context and independent of context to ensure mastery of decoding skills.

## MA $\quad$ Rating: 4

Research in reading is clearly used to inform the acquisition of decoding skills, and standards expect use of real and nonsense words in the primary grades to ensure mastery of decoding skills. There is good coverage of key comprehension skills across subject areas, as well as use of meaningful reading materials.

\section*{DMA | Rating: 4 |
| :--- | :--- |}

Research in reading is clearly used to inform the acquisition of decoding skills, with standards that expect application both in context and independent of context in the primary grades to ensure mastery of decoding skills. There is good coverage of key comprehension skills across subject areas, as well as the use of meaningful reading materials.
2. The standards make clear that interpretations of written texts should be supported by logical reasoning, accurate facts, and adequate evidence.

\section*{| CC | Rating: 4 |
| :--- | :--- |}

The standards indicate that interpretations of any text must be consistent with what the author wrote. Evidence is required for interpretations or claims for all texts.
CA $\quad$ Rating: 4

The standards indicate that interpretations of any text must be consistent with what the author wrote. Evidence is required for interpretations or claims for all texts.

## MA $\quad$ Rating: 4

The standards indicate that interpretations of any text must be consistent with what the author wrote. Evidence is required for interpretations or claims for all texts.

## DMA $\quad$ Rating: 4

Evidence is required for interpretations or claims for all texts. E.g., 5.N.5 "Use reasoning to determine the logic of an author's conclusion in a persuasive text and provide evidence from the text to support reasoning." 7.F.3 "Identify the theme of a story or novel, whether stated or implied, using evidence from the text."
3. It expects students to read independently on a daily basis through the grades, and provides guidance about quality and difficulty.

| CC | Rating: 4 |
| :--- | :--- |
| Students are expected to read independently, and guidance is provided in Appendix B on quality and <br> difficulty through the grades. However, there is no indication of who chose the titles in grades K-8 and <br> whether they were independently vetted by literary experts, as were authors/titles in Appendices A and B in <br> the Massachusetts 2001 and 2010 ELA documents. |  |
| CA $\quad$ Rating: 4 |  |
| Independent reading is encouraged, quantity is spelled out per grade, and the document refers to lists of <br> titles in an accompanying document. |  |
| MA $\quad$ Rating: 4 |  |
| Independent reading is encouraged in Guiding Principle 3, and the quality of the reading is indicated in <br> graded lists of chiefly authors in Appendices A and B. |  |
| DMA | Rating: 4 |
| Guiding Principle 3 encourages independent reading, and two graded lists of chiefly authors in Appendices <br> A and B provide guidance on quality and difficulty. |  |

## B. Value of Literary Study

1. The standards enable English teachers to stress literary study more than informational reading at the secondary level.

| CC | Rating: $\mathbf{1}$ |
| :--- | :--- |
| Nonfiction or informational reading has been weighted more than imaginative literature in ELA at all grade <br> levels- <br> level). This ten standards for the former and nine for the latter at all grade levels (not just at the elementary |  |
| CA | Rating: $\mathbf{3}$ |
| There are about the same number of standards for literary reading as for non-literary reading at all grade <br> levels, so literary study is unlikely to be weighted more through the grades. Moreover, since elementary <br> teachers in self-contained classes usually teach reading in all content areas as well the ELA class, <br> informational reading is apt to be taught more than literary reading in K-6 at present. |  |
| MA | Rating: $\mathbf{4}$ |
| There are more standards for literary study than for informational reading at all grade levels. This helps to <br> balance that fact that elementary teachers in self-contained classes usually teach reading in all content areas <br> as well the ELA class so that informational reading is already apt to be taught more than literary reading in <br> K-6. |  |
| DMA | Rating: 4 |
| There are more standards for literary study than for informational reading at all grade levels. This helps to <br> balance that fact that elementary teachers in self-contained classes usually teach reading in all content areas <br> as well the ELA class so that informational reading is already apt to be taught more than literary reading in <br> K-6. |  |

2. The document indicates that what students are assigned to read should be chosen on the basis of literary quality, cultural and historical significance, or potential to promote understanding of more complex ideas and language.

| CC | Rating: $\mathbf{4}$ |
| :--- | :--- |
| Yes. See the sidebar note on p. 35 and Appendix B. |  |
| CA | Rating: $\mathbf{1}$ |
| There is no clear statement to the effect that all assigned texts should be chosen for literary quality, cultural <br> and historical significance, or intellectual growth. |  |
| MA | Rating: $\mathbf{4}$ |
| Guiding Principle 3 and Appendices A and B make it clear that the English curriculum should draw on texts <br> that have intellectual, cultural, historical, civic, and literary merit at all educational levels. |  |
| DMA | Rating: $\mathbf{4}$ |
| Guiding Principle 3 and Appendices A and B make it clear that the English curriculum should draw on texts <br> that have intellectual, cultural, historical, civic, and literary merit at all educational levels. |  |

3. The document promotes study of American literature.

\section*{| CC | Rating: 3 |
| :--- | :--- |}

It does so only in grades 11/12. It is not mentioned in earlier grades where it would be appropriate (e.g., American folktales, American tall tales)
CA $\quad$ Rating: 3

It is mentioned in the standards only for grades 11-12.
MA $\quad$ Rating: 4
American literature is described in an inclusive way with illustrative works and authors in Appendix A and Appendix B.
DMA $\quad$ Rating: 4
American literature is described in an inclusive way, with illustrative works and authors in Appendix A and Appendix B, with examples of titles or authors for high school standards, and for specific genres (e.g., American folktales in grade 2; American tall tales in grade 4).

## C. Organization and disciplinary coverage of the standards

1. They are grouped in categories and subcategories reflecting coherent bodies of scholarship or research in reading and the English language arts.

| CC | Rating: $\mathbf{3}$ |
| :--- | :--- |
| The "college and career readiness anchor standards" governing the grade-level standards are not as a group <br> internationally benchmarked or supported by research evidence or scholarship. There is no evidence for <br> the effectiveness of a skills-based framework for grade-level standards. The ten CCRAS for Reading are <br> organized under an incoherent group of categories. In contrast, the grade-level standards are organized under <br> a coherent group of categories, with major subcategories under each whose titles for the most part reflect <br> coherent bodies of scholarship or research. |  |
| CA | Rating: $\mathbf{4}$ |
| Major categories and subcategories reflect coherent bodies of scholarship or research. |  |
| MA | Rating: $\mathbf{4}$ |
| Major categories and subcategories reflect coherent bodies of scholarship or research. |  |
| DMA | Rating: $\mathbf{4}$ |
| The 2001 <br> reflect coherent bodies of scholarship or research in reading and ELA. |  |

2. The standards clearly address listening and speaking. They include use of various discussion purposes and roles, how to participate in discussion, desirable qualities in formal speaking, and use of established as well as peer-generated or personal criteria for evaluating formal and informal speech.

## CC $\quad$ Rating: 3

They address most of these elements but do not address the use of established criteria for evaluating formal and informal talks, presentations, or speeches.

## CA $\quad$ Rating: 3

Most of the above areas are adequately covered. Standards need to include more about the different purposes of speech (e.g., conversation, discussion, formal presentations, debate, oratory, improvisation) and systematically increase the expectations for each grade level starting in K. They do not address the use of established criteria for evaluating formal and informal talks, presentations, or speeches.

## MA $\quad$ Rating: 3

Most of the above areas are adequately covered. Standards need to include more about the different purposes of speech (e.g., conversation, discussion, formal presentations, debate, oratory, improvisation) and systematically increase the expectations for each grade level starting in K.

## DMA $\quad$ Rating: 4

These standards go far beyond what was in the 2001 document to highlight participatory knowledge and the listening and speaking skills needed for civic engagement.
3. The standards clearly address reading to understand and use information through the grades. They include progressive development of reading skills, knowledge and use of a variety of textual features, genres, and reading strategies for academic, occupational, and civic purposes.

| CC | Rating: $\mathbf{3}$ |
| :--- | :--- |
| The standards clearly address reading to understand and use information through the grades. They do not <br> clearly distinguish modes of organization from structural elements of an expository text and misinform <br> elementary teachers. |  |
| CA | Rating: $\mathbf{4}$ |
| All of the above areas are adequately covered. |  |
| MA | Rating: $\mathbf{4}$ |
| All of the above areas are adequately covered. |  |
| DMA | Rating: $\mathbf{4}$ |
| All of the above areas are adequately covered. |  |

4. The standards clearly seek to develop strong vocabulary knowledge and dictionary skills.

| CC | Rating: $\mathbf{1}$ |
| :--- | :--- |
| Although the vocabulary standards highlight specific figures of speech and rhetorical devices, they do not <br> teach dictionary skills through the grades, use of glossaries for discipline-specific terms, and some kinds <br> of words that must be taught (e.g., foreign words used in written English that do not appear in an English <br> language dictionary). Common Core leans heavily on use of context to determine the meaning of unknown <br> words but provides no standards on different ways to teach use of context for this purpose. In addition, one <br> key standard contains an inaccurate description and examples of the difference between the connotative and <br> denotative meaning of a word. ${ }^{16}$ |  |
| CA | Rating: $\mathbf{3}$ |
| Although vocabulary standards highlight specific figures of speech and rhetorical devices, there are no <br> standards for dictionary skills or use of a glossary for technical or discipline-specific terms. |  |
| MA | Rating: $\mathbf{4}$ |
| The standards develop dictionary skills through the grades, pay attention to many different kinds of <br> vocabulary groups at each educational level, and teach ways to use context. |  |
| DMA | Rating: $\mathbf{4}$ |
| The standards develop strong vocabulary knowledge and dictionary skills from K-11, clarify the use of <br> glossaries for technical and discipline-specific terms, and teach ways to use context |  |

5. The standards clearly address the reading, interpretation, and critical evaluation of literature. They include knowledge of diverse literary elements and genres, use of different kinds of literary responses, and use of a variety of interpretive and critical lenses. They also specify those key authors, works, and literary traditions in American literature and in the literary and civic heritage of English-speaking people that all students should study because of their literary quality and cultural significance.

| CC | Rating: $\mathbf{1}$ |
| :--- | :--- |
| Most of the above areas are covered but very unsystematically. Most literature standards lack examples <br> of authors, works, literary traditions, and literary periods. Only a few standards indicate specific cultural <br> content (at grades 11-12) and only a few examples are given in Appendix B. |  |
| CA | Rating: $\mathbf{3}$ |
| Almost everything is covered except for key authors or works and specifics about reading level at each <br> grade. A few standards in grades 11 <br> American literature as well as of literary periods in other traditions. Reference is made to lists of works in <br> another document, but the lists vary widely in reading levels. |  |
| MA | Rating: $\mathbf{3}$ |
| All formal aspects of literary study are covered. Appendix A and Appendix B contain a list of key authors <br> or works in American literature, British literature, and world literature, both contemporary and past, to <br> outline the substantive content of the English curriculum through the grades. Guiding Principle 3 and <br> the introduction to the Reading and Literature Strand indicate that the "substantive content of English <br> language arts programs should be derived in large part from these appendices." However, the standards by <br> themselves do not point to key groups of authors, works, literary periods, and literary traditions to outline <br> more precisely the actual content of the high school English curriculum. |  |
| DMA | Rating: $\mathbf{4}$ |
| The literature standards distinguish through sub-strands the major types of imaginative literature (poetry, <br> drama, fiction, and traditional literature) and cover all formal aspects of literary study. Appendix A and |  |
| Appendix B contain lists of key authors in American, British, and world literature, contemporary and past, <br> to outline the substantive content of the English curriculum through the grades, especially at the high school <br> level, and Guiding Principle 3 points to their use. |  |

6. The standards clearly address writing for communication and expression. They include use of writing processes, established as well as peer-generated or personal evaluation criteria, and various rhetorical elements, strategies, genres, and modes of organization.

\section*{| CC | Rating: 3 |
| :--- | :--- |}

Most aspects of writing are addressed well. But there is nothing on the use of established or peer-generated criteria for evaluating writing or written presentations. The sub-strand on "argument" confuses argument with expression of opinion in the elementary grades, confuses academic argument with persuasive writing throughout, and doesn't clarify the key concepts of persuasive writing: purpose and audience. There is no scholarship from Aristotle or Brooks and Warren to Kinneavy to support these three "types" of writing as they are described and thus this strand badly misinforms ELA teachers throughout the grades.

| CA | Rating: $\mathbf{3}$ |
| :--- | :--- |
| Most aspects of writing are addressed well. But there is nothing on the use of established or peer-generated <br> criteria for evaluating writing or written presentations. |  |
| MA | Rating: $\mathbf{4}$ |
| All aspects of writing are addressed well. There are standards on the use of established criteria as well as <br> peer-generated criteria for evaluating writing. |  |
| DMA | Rating: $\mathbf{3}$ |
| Most aspects of writing are addressed well. But there is nothing on the use of established or peer-generated <br> criteria for evaluating writing or written presentations. |  |

7. The standards clearly address oral and written language conventions. They include Standard English conventions for sentence structure, spelling, usage, penmanship, capitalization, and punctuation.

| CC | Rating: 1 |
| :--- | :--- |

Oral and written language conventions are addressed, but the vertical progressions don't always make sense, many standards are placed at inappropriate grade levels, and much of the linguistic terminology is inappropriate at the grade level it appears (e.g., grade 2: "Use adjectives and adverbs, and choose between them depending on what is to be modified." Or in grade 4: "Use modal auxiliaries to convey various conditions."
CA $\quad$ Rating: 4
All of the above areas are adequately covered and in student- and teacher-friendly language.

| MA | Rating: 4 |
| :--- | :--- |

All of the above areas are adequately covered and in student- and teacher-friendly language.
DMA $\quad$ Rating: 4
All of the above areas are adequately covered and in student- and teacher-friendly language.
8. The standards clearly address the nature, dynamics, and history of the English language. They include the nature of its vocabulary, its structure (grammar), the evolution of its oral and written forms, and the distinction between the variability of its oral forms and the relative permanence of its written form today.

| CC | Rating: $\mathbf{3}$ |
| :--- | :--- |
| There is nothing on the distinctions among oral dialects or between oral and written forms of English, or on <br> the history of the English language. Standards on word origins and etymologies are useful but need to be <br> accompanied by standards teaching dictionary skills. |  |
| CA | Rating: $\mathbf{4}$ |
| In grade 8, a vocabulary standard expects students to "understand the most important points in the history of <br> the English language and use common word origins to determine the historical influences on English word <br> meanings." Good use of morphology to illustrate word meaning and development.t |  |
| MA | Rating: 4 |
| All of the above areas are addressed specifically in the standards for Language Study. |  |
| DMA | Rating: $\mathbf{4}$ |
| These aspects of the English language are addressed in the strands for Vocabulary and Concept <br> Development and for Formal and Informal English, especially in relation to the origin of English words, the <br> many foreign words used in written English, and the oral dialects used in literature and in various regions of <br> the country and their differences with standard oral and written English. |  |

9. The standards clearly address research processes, including developing questions and locating, understanding, evaluating, synthesizing, and using various sources of information for reading, writing, and speaking assignments. These sources include dictionaries, thesauruses, other reference materials, observations of empirical phenomena, interviews with informants, and computer data bases.

| CC | Rating: $\mathbf{4}$ |
| :--- | :--- |
| All of the above areas seem to be adequately covered although it is not clear. |  |
| CA | Rating: $\mathbf{4}$ |
| All of the above areas are adequately covered. |  |
| MA | Rating: $\mathbf{4}$ |
| There are grade-level standards outlining specific developmental objectives across educational levels that <br> address the phases and components of the research process. |  |
| DMA | Rating: $\mathbf{4}$ |
| There is a complete strand on the research process outlining specific developmental objectives across <br> educational levels that address the phases and components of the research process. |  |

## D. Quality of the standards

1. They are clear, specific, and measurable (i.e., they can lead to observable, comparable results across students and schools).

\section*{| CC | Rating: 3 |
| :--- | :--- |
| The |  |}

There are many vague standards with unclear meanings and inconsistently interpretable meanings. E.g., "Compare and contrast the structure of two or more texts and analyze how the differing structure of each text contributes to its meaning and style." What kind of texts does the writer have in mind? What will be learned if the texts address very different topics? E.g., "Analyze a particular point of view or cultural experience reflected in a work of literature from outside the United States, drawing on a wide reading of world literature." How much reading of world literature must precede the reading of a specific work that is to be analyzed for the author's point of view? Thus, only some standards are measurable as is. There are also many standards with inappropriate or misleading examples, e.g., Informational reading standard 9, grade 6: "Compare and contrast one author's presentation of events with that of another (e.g., a memoir written by and a biography on the same person)." This standard needs a sensible example. Moreover, in the primary grades, some standards require teachers to prompt or observe without specifying what would constitute meeting the standard.

| CA | Rating: 4 |
| :--- | :--- |
| Overall, they are clear, specific, interpretable, teachable, measurable, and reliably rated. |  |
| MA | Rating: 4 |
| Overall, they are clear, specific, interpretable, teachable, measurable, and reliably rated. |  |
| DMA | Rating: 4 |
| Overall, they are clear, specific, interpretable, teachable, measurable, and reliably rated. |  |

2. They are of increasing intellectual difficulty at each higher educational level and cover all important aspects of learning in the area they address.

\section*{| CC | Rating: 3 |
| :--- | :--- |}

Many standards do not show meaningful increases in intellectual difficulty over the grades. K-2 standards are weak due to overuse of "prompted" or "unprompted" as the key feature of difficulty; prompted learning indicates the level of independence associated with student performance. In addition, many grade-level standards are simply paraphrases or repetitions of the governing CCRS, especially in grades 6-8. It is not clear why grades 6-7 contain standards calling for significant amounts of time to be devoted to comparing literary works to movies and staged productions.

| CA | Rating: 3 |
| :--- | :--- |

Most of the focused standards show meaningful increases in difficulty over the grades and address the important aspects of learning in the area, although it is not clear how demanding the standards in grades 10 and 11 really are without specific grade levels or titles of key works to illustrate a grade level.

## MA $\quad$ Rating: 4

The standards in each strand show meaningful increases in difficulty and/or complexity through the grades related to skill development.

## DMA $\quad$ Rating: 4

The standards in each strand show meaningful increases in difficulty and/or complexity through the grades related to skill development.
3. They index or illustrate growth through the grades for reading by referring to specific reading levels or to titles of specific literary or academic works as examples of a reading level.

| CC | Rating: 4 |
| :--- | :--- |

While the reading and literature standards only occasionally provide examples of specific texts or authors, Appendix B contains a long list of illustrative titles for each grade for the main genres.

## CA $\quad$ Rating: 3

The kinds of literary techniques taught often suggest complex texts, but no authors or works or reading levels are given (except for the mention of Macbeth). Reference is made to lists of titles in another document, but the lists are very long and include a wide range of reading levels.

## MA $\quad$ Rating: 4

The reading and/or literature standards are almost always accompanied by examples of specific texts and/or authors. The examples are stronger at higher grade levels. There are also two appendices with recommended lists of authors divided into educational levels.

## DMA $\quad$ Rating: 4

The standards do both, especially at the high school level. Appendices A and B contain lists chiefly of authors divided into grade spans from K-12. The high school reading and literature standards contain general statements indicating difficulty level (e.g., classical Greek plays, or classical epic poetry) or list specific authors and/or titles as examples.
4. They illustrate growth expected through the grades for writing with reference to examples and rating criteria, in the standards document or in other documents.

\section*{| CC | Rating: 1 |
| :--- | :--- |}

Appendix C is a collection of annotated student writing samples at all grade levels. However, no rating criteria, say, on a 1 to 6 scale, are offered by grade level-a serious and puzzling omission. Based on the annotations and the compositions themselves, it is clear what the best and least developed compositions are. But it is not at all clear how teachers are to develop common expectations for somewhat above grade-level, about grade-level, and somewhat below grade-level performance at a particular grade level (i.e., for most of the age cohort).

## CA $\quad$ Rating: 4

Criteria and examples of student essays appear on the department's website.

## MA $\quad$ Rating: 4

Writing samples and the criteria for rating them are maintained on the Department's website in connection with its testing program.

## DMA $\quad$ Rating: 4

Writing samples and the criteria for rating them are maintained on the Department's website in connection with its testing program.
5. Their overall contents are sufficiently specific, comprehensive, and demanding to lead to a common core of high academic expectations for all students in the state.

\section*{| CC | Rating: 1 |
| :--- | :--- |}

Common Core's relatively content-empty reading and literature standards cannot lead by themselves to a common core of high academic expectations. They cannot frame an academic curriculum. The basic work will have to be done at the local level unless the testing consortia pre-empt curriculum decision-making at the local level in order to develop test items with a knowledge base to which skills can be applied.
CA $\quad$ Rating: 3

For the most part. California's standards require study of literary traditions in American literature as well as of literary periods in other traditions. However, because they do not specify key groups of works and authors that outline essential substantive content for the high school English curriculum, they can be interpreted in different ways by teachers and test developers and thus fail to lead to a common core of high academic expectations for all students.

| MA | Rating: 4 |
| :--- | :--- |

Yes, these standards can lead and have led to a common core of high academic expectations.
DMA $\quad$ Rating: 4

These standards can lead to a common core of higher academic expectations than the 2001 standards.

## E. Summary

Table 1: Average Points per Section and Total Average for Four ELA Documents

|  | CC | CA | MA | DMA |
| :---: | :---: | :---: | :---: | :---: |
| Reading Pedagogy and Independent Reading | 3.6 | 3.6 | 4 | 4 |
| Value of Literary Study | 3 | 2.3 | 4 | 4 |
| Organization and Disciplinary Coverage of the Standards | 2.4 | 3.5 | 3.7 | 3.8 |
| Quality of the Standards | 2.5 | 3.5 | 4 | 4 |
| Total | 2.7 | 3.4 | 3.9 | 3.9 |

## IV. Review of Mathematics Standards

The National Mathematics Advisory Panel (NMAP) was established in 2006 to use the "best available scientific evidence" for recommendations on ways "to foster greater knowledge of and improved performance in mathematics among American students" (p. xiii). ${ }^{17}$ Its first charge was to outline: "the critical skills and skill progressions for students to acquire competence in algebra and readiness for higher levels of mathematics" (p. 71). The Panel proposed three clusters of concepts and skills: Fluency with Whole Numbers, Fluency with Fractions, and Particular Aspects of Geometry and Measurement. For this review, we compare how Common Core's mathematics standards address one or two key topics in each cluster with the way those topics are addressed by California's current mathematics standards (approved in 1997), by the 2010 draft standards in Massachusetts, and by standards used in high-achieving countries. We then compare how each of these documents addresses preparation for Algebra I in grade 8 and high school mathematics.

## A. Fluency with Whole Numbers

The NMAP's final report recommends the following two grade-level benchmarks (goals) for fluency with whole numbers (p. 20). (NMAP recommendations are in blue italics throughout this section.)

> - By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
> - By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

Massachusetts expects students to achieve fluency with addition and subtraction using the standard algorithms by grade 3. Both California and Common Core expect students to achieve
fluency with addition and subtraction using the standard algorithms by grade 4 .

Both California and Massachusetts expectstudents to reach fluency with multiplication using the standard algorithm by grade 4 , and fluency with division using the standard algorithm by grade 5. In contrast, Common Core expects fluency with multiplication using the standard algorithm by grade 5 , and fluency with division using the standard algorithm by grade 6 .

Use of the standard algorithms requires commitment to memory of addition facts for addition and subtraction, and the multiplication table for multiplication and division. Both California and Massachusetts require memorization of addition facts to 20 in grade 1, while Common Core expects memorization to 18 by grade 2 . California expects memorization of the multiplication table by grade 3, while both Massachusetts and Common Core expect it in grade 4.

In summary, all three sets of standards develop fluency with the arithmetic of integers by grade 5 as recommended by the NMAP, with the notable exception of division, which Common Core defers to grade 6 . This is not a small matter for two reasons. First, grade 6 is commonly the first year of middle school in this country, and deferring fluency with division to grade 6 removes responsibility for complete fluency of operations with integers from the K-5 elementary school. Common Core's deferral of the division algorithm to grade 6 creates the potential for underdeveloping the foundations of division in elementary schools that lack accountability for this skill. Second and more important, grade 6 in Common Core focuses on the development of a major mathematical topic: ratios (and ratio reasoning like rates and use of percents). Deferring learning of the division algorithm to grade 6 means that students tackle this new and demanding concept without complete fluency with division, which may undermine their ability to learn the new concepts of ratio and proportional thinking.

In addition, both Massachusetts and California developestimation skills in parallel to development of computation skills, to assist in estimating the expected result of an operation and to verify its correctness. Both California and Massachusetts start developing this skill from kindergarten on. ${ }^{18}$ In contrast, Common Core defers the initial development of this skill to grade 3. As a result, students in grade 6 may end up with insufficient experience to develop the deep sense of numeracy that estimation instills.

## B. Fluency with Fractions

Much praise has been directed to the careful and consistent way that fractions are developed in Common Core's standards. And, indeed, its treatment of common fractions is lengthy and meticulous, even if steeped in pedagogy ${ }^{19}$ and more reminiscent of a lesson plan than of content standards. Yet few have noted that Common Core starts developing decimal fractions (decimals) in grade 4, two years behind both California and Massachusetts. Or that it fails to introduce decimals using dollar bills and coins, a traditional and effective way to introduce decimals that builds on the knowledge and natural fascination of children with money.

In contrast, the first two NMAP benchmarks for fractions expect students to recognize their two common representations in grade 4 and to develop fluency in conversions between them.

- By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.

Both California (4NS1.9) and Massachusetts (4.N.5) meet this expectation in grade 4. Common Core, however, does not discuss conversion or relations between common fractions and decimals until grade 7.

[^2]
## decimals and common percent and with the addition and subtraction of fractions and decimals.

Similarly, both California (5NS1.2, 5NS1.5, 5NS2.1) and Massachusetts (5.N.2, 5.N.3, 5.N.7, 5.N.8) meet this expectation, with Massachusetts deferring study of percents to grade 6 (6.N.5). In contrast, not only does Common Core defer conversion between decimals and fractions to grade 7 , it also defers the relation and conversion of percents, introduced in grade 6 , to other forms of fractions to grade 7 .

- By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.

California meets this benchmark by grade 6 (6NS2.3), while Massachusetts meets most of it in grade 6 (6.N.5, 6.N.12, 6.NS.16), deferring fluency for multiplication and division with negative numbers to grade 7 (7.N.1, 7.N.4). In contrast, Common Core introduces only the concept of a negative number in grade 6, and expects fluency with negative numbers in grade $7 .{ }^{20}$

In summary, while Common Core has a strongly developed learning progression for common fractions, it fails to build on money to introduce decimal fractions early. More important, it completely neglects development of flexibility with fraction representations, a focus of NMAP concern. Flexibility with fraction representations is also of cardinal concern in Adding It Up and the National Council of Teachers of Mathematics (NCTM)'s Curriculum Focal Points. ${ }^{21}$

Another surprising deficiency in Common Core's approach to fraction study, despite the prominent attention it gives to fractions and the wide praise its approach has received, is the absence of a material on teaching and using least common denominators beyond the simple multiplication of denominators. While conceptually unimportant, this neglect dooms students to handle unwieldy numbers even when adding simple fractions such
as $1 / 6+5 / 18+2 / 3$, a situation that is bound to affect their fluency and comfort with operations on fractions. Common Core also neglects teaching prime factorization ${ }^{22}$ at any grade and consequently develops only rudimentary skills in finding least common multiple or greatest common factor. Both California and Massachusetts extensively develop these skills, which become critical in later grades for fluency in manipulating polynomial and rational expressions.

Finally, the coherence of fraction study in Common Core is seriously undermined by the repeated use of visual fractions (number lines, area models, tape diagrams). R. James Milgram's testimony to the California Academic Content Standards Commission on July 7, 2010, provides further details on the inappropriateness of this practice and the confusion it will engender (see Appendix B in this report). ${ }^{23}$

## C. Particular Aspects of Geometry and Measurement

For geometry and measurement, the NMAP recommends the following two benchmarks in grades 5 and 6:

- By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
- By the end of Grade 6, students should be able to analyze the properties of twodimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.

All three sets of standards we are reviewing deal with the geometry of two- and three-dimensional bodies in a rather disorganized and often unspecific way, with California's standards the clearest of the three. Common Core focuses on the relations
between rectangular areas and multiplication throughout grade 6 , while both California and Massachusetts make this connection during development of multiplication in the early grades before focusing on the geometrical nature of area. Interestingly, California is explicit about recognizing special triangles like isosceles, equilateral, and right angle in grade 3 , deriving the area of a triangle in grade 5, the sum of triangle angles in grade 5, and area and perimeter of a circle in grade 6. Massachusetts teaches special triangles in grade 4 but doesn't mention the derivation of the area of a triangle (although it expects calculating the surface area of a pyramid in grade 5, which implies student learn at least the formula by then), doesn't teach the sum of triangle angles, but does teach area and perimeter of a circle in grade 6. Common Core doesn't mention special triangles except right triangle in grade 4, doesn't seem to teach how to calculate the area of a triangle at all (except possibly by a rote formula at some unspecified time), but expects them to be known in grade 6. It doesn't teach the sum of triangle angles, but it teaches area and perimeter of circles as late as grade 7 .

The NMAPrecommends the following benchmark for geometry and measurement in grade 7:

- By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

California teaches similarity in grade 6 (6NS1.3) but without explicit connection to slopes, a topic that is covered in its Framework instead. Massachusetts teaches similarity in grade 7, and it may imply connection to the slope of a line:
7.G.3. Apply the geometric concepts of similarity and congruence; relate similarity to concepts of proportionality and scale factor, and use them in solutions of problems. For example, indirectly measure the height of a tree by using its shadow.

Common Core addresses this benchmark directly, but in grade 8-a year later than NMAP recommends:
8.EE.6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $\mathrm{y}=\mathrm{mx}$ for a line through the origin and the equation $y=m x+b$ for a line intercepting the vertical axis at b .

While both California and Massachusetts teach use of the straightedge and compass in K-8 (in grades 5 and 7 in California, and in grade 8 in Massachusetts), as well as some classical geometrical constructions to develop intuition and an esthetic sense about geometry, Common Core teaches use of only the ruler and protractor in K-8, relegating geometry to just another facet of measurement.

Much more problematic is Common Core's effort to replace the traditional foundations of school geometry. Common Core attempts to base congruency in grade 8 and high school on the notion of rigid motions, and to derive the rest of geometry from this basis. This approach has not been widely used anywhere in the world, and the only known experience with it is considered a failure, as noted on p. 8 in R. James Milgram's testimony to the California Academic Content Standards Commission in July 2010. Perhaps the best assessment of this issue is Milgram's comment in his testimony: "I feel that we are dealing with an experiment on a national scale" (p.10).

## D. Preparation for the Study of Algebra

One of NMAP's major recommendations was a call to prepare more students for an authentic Algebra I in grade 8. "All school districts should ensure that all prepared students have access to an authentic algebra course - and should prepare more students than at present to enroll in such a course by Grade $8 .{ }^{24}$

This recommendation is supported in the report by NMAP's Task Group on Conceptual Knowledge and Skills. The Task Group researched the question of the importance of taking algebra early and found six studies that met NMAP's rigorous criteria for evidence. It summarizes them as follows:

It is important to note that these six studies drew on four national data sets. Three analyzed LSAY data (Ma, 2000, 2005; Wilkins \& Ma, 2002), two used HS\&B data (Smith, 1996; Jones et al., 1986), and one used data from NELS: 88 and the High School Effectiveness Study (Lee et al., 1998). The consistency of their findings is striking. The studies by Ma and others provide some evidence that there are long-term benefits for Grade 7 or 8 students with the requisite mathematical background for algebra if they can take an authentic Algebra course in Grade 7 or 8: higher mathematics achievement in high school and the opportunity to take advanced mathematics course work in Grade 11 or $12 .{ }^{25}$

Of the three sets of standards, only California sequences its content to enable all students to prepare for Algebra I in grade 8. In contrast, both Massachusetts and Common Core aim for only a little more than pre-algebra in grade $8 .{ }^{26}$ However, all high achieving countries teach either the equivalent of Algebra 1 in grade 8 or a combination of Algebra 1 and Geometry in grades 8 and 9. ${ }^{27}$

Common Core's preparation for taking Algebra I in grade 8 is weak. High achieving countries introduce preparatory material in the early grades. In Singapore, grade 5 students do rate problems involving motion at constant speed. Such problems cannot appear in Common Core before grade 6 because its standards on ratio are first taught in grade 6 .

High achieving countries, as well as California and Massachusetts, also begin to solve simple
linear equations in the early grades. For example, California provides in grade 3:

- Represent relationships of quantities in the form of mathematical expressions, equations, or inequalities"
- Express simple unit conversions in symbolic form (e.g., __ inches $=\ldots$ feet $\times 12$ ).
- Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4 s or by multiplying the number of horses by 4).
and in grade 4:
- Use letters, boxes, or other symbols to stand for any number in simple expressions or equations(e.g.,demonstrateanunderstanding and the use of the concept of a variable).
- Understand that an equation such as $\mathrm{y}=3 \mathrm{x}+$ 5 is a prescription for determining a second number when a first number is given.
- Know and understand that equals added to equals are equal.
- Know and understand that equals multiplied by equals are equal.
and in grade 5:
- Know and use the distributive property in equations and expressions with variables.
- Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.

In contrast, while some Common Core standards in grades 1 and 2, e.g., 1.OA(1) and 2.OA(1), appear to be related to this topic, they are unclear. They seem to be asking students to solve simple equations, but the language needs to be clearer for us to be sure, particularly since a grade 6
standard calls for what seems like a more basic concept: (6.EE.2.a.) "Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation 'Subtract y from 5' as $5-\mathrm{y}$."

In addition to a lack of clarity in what Common Core provides in the early grades, there is a marked paucity of standards developing early algebraic skills. Unit conversion doesn't start until grade 4, and when it does, the standards are verbose and unclear:
4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in . Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...
4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

It is hard to know whether these standards expect students to work with expressions or to fill up conversion tables. These standards are particularly difficult to interpret because what is in grade 5 seems less demanding. ${ }^{28}$
5.MD.1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m ), and use these conversions in solving multi-step, real world problems.

Common Core's grade 8 standards also contribute to incoherence. As part of preparation for algebra, Common Core inserts four unrelated statistics standards. They can do little more than develop "statistics appreciation" because students have no foundation to address them mathematically. For example, students are in effect requested to draw a linear best-fit line after "eyeballing" scatterplot data. They are also asked to form hypotheses about the relationships among disparate data sets without the mathematical means to support their guesses. These standards appear to contradict Common Core's goal of "fewer, clearer, higher" standards. ${ }^{\text {ii }}$

## E. Important Issues in Using Common Core's High School Mathematics Standards

1. Its low academic expectations for college and career readiness. Common Core currently marks certain high school mathematics standards with a plus sign ( + ), explaining that "all standards without a $(+)$ symbol should be in the common mathematics curriculum for all college and career ready students." It immediately goes on to state that "standards with a $(+)$ sign may also appear in courses intended for all students," thus leaving it unclear whether or not college readiness includes these standards, too. Common Core implies that the content of its standards is sufficient for about three years of high school mathematicsAlgebra 1, Geometry, and an additional year of mathematics somewhat similar to Algebra II. ${ }^{29}$ But even with the "plus" standards, the content described in its high school mathematics standards lacks some of the content expected for students finishing a typical Algebra II course-the criterion for determining "college readiness" for most four-year colleges in this country. ${ }^{30}$ Without the standards marked "plus," Common Core's content for college readiness is far below what is expected today for college eligibility. ${ }^{31}$

Concerns about the standards that Common Core uses to define college readiness are pointedly expressed inthe Report ofthe Mathematics Review Panel on the Common Core and Massachusetts

Standards, July 2010. ${ }^{32}$ This report was one of the documents that the commissioner of education used for his recommendation to the Massachusetts Board of Elementary and Secondary Education that it vote on July 21, 2010, to adopt Common Core's standards. Written by seven mathematics educators chosen by the commissioner, the Report noted that the 2010 draft Massachusetts standards "have clearer expectations for linear functions and geometric proof while [Common Core's] include a good deal more statistics." The report then expressed concern about Common Core's standards generating "two tracks" of students. It noted that some of the standards marked ( + ) in Common Core are included in the state's draft Algebra, Geometry, and Algebra II courses, e.g., "logarithmic functions, inverse functions, and some aspects of complex numbers." Concerned that the ( + ) standards in Common Core "actually generate two tracks and "that students may be excluded from advanced mathematics courses/ concepts through tracking," the panel suggested that the "Department of Elementary and Secondary Education consider each ( + ) standard individually and determine whether or not, for this state, each ( + ) standard be mandated for inclusion in the Algebra-Algebra II sequence." Clearly, Common Core's standards for "college readiness" in mathematics are far below the state's expectations.

## 2. Their lack of organization in course form.

 When the first draft of Common Core's College and Career Readiness Standards was released in September 2009, many commented on its low aspirations, problematic wording, and organization. After that, the high school standards underwent much rewriting and reorganization. Despite all the drafts, they still ended up in a difficult-to-use form for teachers and textbook publishers. The high school standards are currently grouped into so-called "conceptual categories." Originally there were six, but one-modeling-has since been distributed across the other five. ${ }^{33}$[^3]In contrast, the 2010 draft of Massachusetts' standards (as did its 2000 standards document) organizes high school mathematical content not only by two-year grade spans (9-10 and 11-12), but also by course-Algebra I, Geometry, Algebra II, and Precalculus. California organizes all its high school mathematics standards by course, from Algebra to Calculus.
3. Negative consequences of the failure to organize high school standards by course. The fact that Common Core's high school standards were not written with the structure of high school courses in mind has several negative consequences. First, the depth with which a topic should be treated depends on the course it is in, and this depth is not apparent when the topic appears simply in a long list of topics in a conceptual category. Many mathematical topics appear in multiple courses. For example, manipulation of polynomials, their factorization, and manipulation of rational expressions appear in Algebra 1, Algebra 2, trigonometry, and mathematics analysis. At the 30,000 feet level, the topic is the same-"the manipulation of X." When one considers course structure while writing standards, one tunes the wording to the required depth for a particular course, e.g., polynomials of second degree, polynomials of trigonometric functions, etc. But when one simply generates a comprehensive list of topics as Common Core does, one ends up with a standard like:

A-APR-7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

This standard is potentially applicable to Algebra 1, Algebra 2, Trigonometry, or Mathematical Analysis. Which shall it be? There is little here to guide teachers or textbook publishers. All depends onits interpretation, which isabsent. Consequently, one may end up replicating it verbatim in four different courses and supplementing it with an explanation, or relying on a teacher to interpret
it properly when it is to be taught. Neither is efficient or necessarily effective, and Common Core has many such examples.

In addition, as they now stand, Common Core's high school mathematics standards can be used by the states in varying ways to construct high school mathematics courses, with no uniformity in these courses across states. Textbook publishers, too, can construct high school mathematics courses in varying ways and at different depths, undermining the primary reason for creating common standards in the first place.
4. A tendency to undervalue technical skills. For example, while Common Core's standards mention manipulation of polynomials and rational expressions, there is little specificity as to the level of fluency needed, and the standards omit such basics as factorization skills. In fact, when California considered how to strengthen Common Core's mathematics standards, many of these skills were judged as needing to be added to create an Algebra 1 course comparable to the current one. ${ }^{34}$
5. Missing standards. As we noted earlier, Common Core claims to provide content for about two-years-worth of high school mathematics beyond Algebra 1. Yet many topics typically included are conspicuously absent or barely addressed. For example, here is Common Core's standard on factoring, a key technical skill.

A-SSE-2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$.

In comparison, here are the comparable California and Massachusetts standards:

CA-A1-11. Students apply basic factoring techniques to second- and simple thirddegree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of
two squares, and recognizing perfect squares of binomials.

MA-10.A.9. Demonstrate facility in symbolic manipulation of polynomial and rational expressions by applying the commutative, associative and distributive principles. Include factoring, simplifying rational expressions, and applying the rules of positive integer exponents.

The comparison is striking. Does Common Core's standard address general factoring skills, or does it ask only for recognition of certain "well known" forms? What class of expressions is Common Core's standard intended to be applied to? Teachers will have to guess or let a test writer behind a closed door decide for the nation.

Logarithms are barely mentioned. Mathematical induction is absent. Solving equations and problems with absolute values is missing. Parametric equations are absent. Infinite geometric series are absent. Arithmetic series are simply forgotten. Conic sections are incompletely handled. Frequency and amplitude of periodic functions are mentioned, but their phase was left out. Double-angle formulas are mentioned, but half-angle formulas are absent. Polar coordinates are mentioned, but no work is expected with polar forms of functions. And the list goes on.

## F. Summary

In K-8: All three sets of standards we reviewed develop fluency with the arithmetic of integers by grade 5 as recommended by NMAP, with the notableexceptionofdivision, whichCommonCore defers to grade 6. Although Common Core has a well-developed learning progression for common fractions, it fails to build on money to introduce decimal fractions early, it neglects development of flexibility with fraction representations, it provides no material on teaching and using least common denominators beyond the simple multiplication of denominators, it fails to teach prime factorization at any grade, and it seriously misuses visual fractions.

In 9-12: Common Core's replaces the traditional Euclidean foundations of school geometry with an approach in grade 8 and high school that is based on the notion of rigid motions. This approach has not been widely used elsewhere, has been considered ineffective where it was tried out, and is at best an experiment. In addition, Common Core's preparation for Algebra I in grade 8 is weak compared to the standards in California and in high achieving countries. Finally, there are many weaknesses in Common Core's high school standards. Compared to the standards in California and Massachusetts for Algebra I, Geometry, and Algebra II, the content of Common Core's standards for these courses show low academic expectations for "college readiness." Moreover, its high school standards are not organized into courses or by grade level or grade span, as they are in California, Massachusetts, and everywhere else, thus providing no clear and uniform guidance to high schools and textbook publishers on course content.

## V. Concluding Remarks on English Language Arts Standards

A. Common Core's literature and reading standards in grades 9-12 do not prepare students for college and career better than those in California and Massachusetts.

Common Core's high school standards fall well short of those in California and in Massachusetts 2001 and 2010 in specificity of literary and cultural content. Common Core provides the right words for the grades 6-12 standards in a sidebar on p. 35: "To become college and career ready, students must grapple with works of exceptional craft and thought whose range extends across genres, cultures, and centuries..." Common Core also provides a lengthy Appendix B containing lists of "Texts Illustrating the Complexity, Quality, and Range of Student Reading," with dozens of titles of important works for each grade level. Nevertheless, while Common Core includes standards requiring reading of a Shakespeare play and seminal works in American literature
in grades 11-12, its grade-level standards do not ensure adequate preparation through the grades for studying these works or sufficient literary and cultural knowledge for authentic collegelevel work. One searches in vain for literary and cultural content through the grades that would lead to these standa

In contrast, in grades 9-10, California expects students to "Identify Greek, Roman, and Norse mythology and use the knowledge to understand the origin and meaning of new words." Massachusetts has an entire K-12 strand in its 2001 and 2010 documents on myth, traditional narrative, and classical literature.

As for American literature, for grades 11-12, California wants a full course in American literary history, setting forth such intellectual objectives as:
a. Trace the development of American literature from the colonial period forward.
b. Contrast the major periods, themes, styles, and trends and describe how works by members of different cultures relate to one another in each period.
c. Evaluate the philosophical, political, religious, ethical, and social influences of the historical period that shaped the characters, plots, and settings.

Massachusetts 2010 goes beyond these general intellectual objectives to provide standards on American poetry, drama, and fiction in all literary periods in grade 11 and on British literary history, drama, fiction, and poetry in grade 12 (as in: "Demonstrate familiarity with major British poets and some of their poems in each period of British literary history"). Both its 2001 and 2010 documents refer to a list of major authors for study of American and British literary history in an Appendix.

Similar differences in cultural specificity appear in the informational reading strand. Common Core does require students to analyze U.S. documents of historical and literary significance
for how they address related themes and concepts in grades 9-10, and to analyze U.S. documents of historical and literary significance in the 17tth, 18th, and 19th centuries for their themes, purposes, and rhetorical features in grades 11/12. But Massachusetts 2010 lays out more coherent progressions of standards (and throughout the grades). It expects seminal U.S. documents in the 19th and 20th centuries to be studied in grade 10 and follows up with such standards in grade 11 as: "Synthesize information from texts written in the 18th or 19th century or before to address ideas in foundational texts written in the 18th or 19th century (e.g., read selections from John Locke's Second Treatise on Government, Montesquieu's Spirit of the Laws, and Madison's Notes on the Constitutional Convention) and trace the history of the ideas presented in the Constitution of the United States." And in grade 12, students are to analyze texts that have worldwide historical and literary significance with respect to their purposes, central arguments, and social, political, and cultural contexts.

In conclusion, by adopting Common Core's standards for their own, California and Massachusetts significantly weaken the intellectual demands on students in the areas of language and literature. They also weaken the base of literary and cultural knowledge needed for actual college-level work now implied by each state's current or draft standards.
B. English teachers cannot readily teach to all of Common Core's literature and reading standards in grades 9-12.

Given what they are prepared to teach based on their undergraduate or graduate coursework, English teachers cannot teach to many of Common Core's informational reading standards and they are unlikely to try to do so. English teachers can clearly teach the literary content required in its $9-12$ standards-one play by Shakespeare, one play by an American writer, and foundational works of American literature from the 18th, 19th, and early 20th centuries in grade 11 or 12.

But Common Core expects English teachers to spend over 50 percent of their time addressing literary nonfiction and informational texts such as seminal U.S. documents and U.S. Supreme Court decisions.

It is important to note that the 2010 draft Massachusetts standards also expect high school English teachers to address seminal U.S. documents and other 18th -century (or earlier) texts. But its nonfiction/informational strand is only one of five literature/reading strands (thus requiring only 20 percent of their time on textbased study, not over 50 percent) and English teachers will be able to select and teach seminal documents they have often addressed (e.g., The Declaration of Independence or Letter from Birmingham Jail)
C. English teachers will likely be held accountable for the results on high school reading tests.

It is likely that English teachers will be held accountable for much more than 50 percent of the results on Reading tests based on Common Core's college and career readiness standards. A clear answer to this question is not possible because Common Core speaks with forked tongue on who will teach students to read informational texts. Its introduction (p. 8) clearly states that history and science teachers are to teach the literacy standards in their subject areas. However, it also notes that its grades 6-12 standards require a "significant amount of reading of informational texts...in and outside the ELAclassroom"(p.5).Moreover, while illustrative titles of informational texts for grades 6-12 in the ELA section of its standards document are all labeled "literary nonfiction"-a comforting label for English teachers-illustrative titles of informational texts for ELA teachers in Appendix $B$ are all labeled "informational"-which makes far more sense than "literary nonfiction" for a list that includes, at grades 11-12, for example, Thomas Paine's Common Sense, the U.S. Bill of Rights, and a book chapter by historian Richard Hofstadter-works that fit uncomfortably under the label of literary nonfiction. Anchor standard
\#7 ("Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.") very clearly implies that ELA teachers are going to be teaching much more than literary nonfiction at every grade level to satisfy Common Core's informational reading requirements. Moreover, Common Core wants reading passages on high school collegereadiness tests to reflect NAEP's distribution of passages for its high school reading tests: 70 percent informational, 30 percent literary (p. 5). It is highly unlikely that English teachers will be held accountable for only a little more than 30 percent of the Reading test score.
D. Common Core's standards make a coherent K-12 ELA curriculum unattainable.

Unlike standards that point to the general cultural or literary knowledge (as well as the generic thinking and language skills) needed at each grade level, Common Core's "anchor" and grade-level standards not only provide no intellectual base or structure for a curriculum, they actually prevent one from emerging. The academic content of English as a K-12 subject area consists of the concepts that guide literary study (including nonfiction) through the grades (e.g., genres, subgenres, rhetorical and literary techniques and elements, literary periods, literary traditions). But the texts that teach these concepts cannot take up even half of the reading curriculum at each grade level if it is to address all of Common Core's reading standards with the weight they requires. What informational topics can contribute to coherent learning progressions from grade to grade in the over 50 percent of the reading at each grade level that is to be informational? What concepts can a progression of informational texts be based on for a coherent English curriculum in grades 6-12? Or is the ELA informational reading curriculum to cannibalize the reading content of the science, history/social studies, mathematics, and arts curricula in grades 6-12, content that English teachers are not expected and prepared to teach?
E. Common Core's ELA standards will require drastic changes in academic, preparation, and professional development programs for prospective or current English teachers.

English teachers will need to take a significant amount of academic coursework (or professional development) in history and political science to understand the historical context, philosophical influences, unique features, and national and international significance, historically and today, of the seminal documents they are being required to teach students how to read. They will also have to undergo professional training in reading scientific and other discipline-based texts. In addition, because the organizing framework for Common Core's ELA standards consists chiefly of generic skills, teachers will need to learn how to teach summary writing, student-generated questioning, paraphrasing, and all the other study skills they have generally never been taught how to teach in their preparation or professional development programs.

## VI. Concluding Remarks on Mathematics Standards

Common Core's project was a laudable effort to shape a national curriculum. Unfortunately, rather than build on the strengths that can be documented in Massachusetts or California, the draft-writers chose to navigate an uncharted path. Consequently, although they sometimes include an interesting treatment of several new ideas that are potentially useful for the development or implementation of a standard, by grade 8 their standards are a year or two behind the National Mathematics Advisory Panel's recommendations, leading states, and our international competitors.

Worse, Common Core's standards impose an experimental geometry curriculum on the nation, without piloting. No discussion has taken place after some experts on the Validation Committee refused to sign off on the standards, and the public has been left with the incorrect impression that mathematicians and other academics
are unified in support of Common Core's mathematics standards. They are not. Common Core's mathematics standards miss chunks of content recommended by the NMAP for K-8, and inexplicably leave large holes in mathematics content currently in the high school curriculum. ${ }^{35}$

## VII. Recommendations

Our analysis of Common Core's mathematics and ELA standards does not support the conclusion drawn by many other reviewers that Common Core's standards provide a stronger and more challenging framework for the mathematics and English language arts curriculum than (or even as equally a challenging framework as) California's current standards and Massachusetts' current (2001) and revised draft (2010) standards do. Common Core's standards will not prepare more high school students for authentic college-level work than current standards in these states do. To the contrary, they may lead to fewer high school students prepared for authentic college-level work. There are many legitimate reasons for having national standards, but they would need to be much stronger than those that Common Core has offered this nation if they are to be worthy of such a role.

1. States adopting Common Core's standards should delete the label of "college and career readiness standards" or "college and career readiness anchor standards" on all of Common Core's standards. Common Core's "college readiness" standards do not point to a level of intellectual achievement that signifies readiness for authentic college-level work. At best, they point to no more than readiness for a high school diploma. States adopting Common Core's standards should recognize the difference and ensure that they do not engage in false advertising. Accepting them at face value will damage our post-secondary institutions as well as the integrity of high school coursework. Adopting states are obligated to use the standards as worded, but there is no legal obligation to use surrounding textual material, appendices, or these labels.

Common Core's standards may well help many states to frame a stronger high school curriculum than their current standards do. Preparing all high school students for a meaningful high school diploma is something we have not yet achieved as a country and still need to do. But it is a far cry from preparing all high school students for authentic college-level work. Thus, preparing some high school sophomores or juniors for credit-bearing freshman coursework in an open admissions post-secondary institution, especially if the coursework has been adjusted downward in difficulty to accommodate them, is a strategy to evade the real problem - how to strengthen the high school academic curriculum to reduce postsecondary remedial coursework in a legitimate way as well as provide satisfactory options to this curriculum for students who do not want or cannot succeed in a traditional academic curriculum or who do not want to go to college.

Pretending that academically less able students are "college-ready" deceives them, their parents, educational institutions (K-12 and postsecondary), and the public at large and debases the meaning of authentic college-level work. The irony of supporting academically less able students in college while their academically more able peers are still finishing high school will eventually not be lost on any of them. It also creates a clear two-class society-those who leave high school to enroll in an open admissions college and those who complete a strengthened high school program to enroll in a more selective college.

Especially in states with more academically rigorous standards and ambitious high school graduation requirements (e.g., four years of high school mathematics), an emphasis on individual advancement and self-fulfillment is a massive shift from, and in a real sense a betrayal of, the principal purpose of K-12 public education-to produce informed and responsible citizens.

## 2. Educational publishers should not develop textbooks and other curricular materials based

on Common Core's ELA standards until several key standards are appropriately placed and/or correctly written and several conceptual errors are corrected. The present set of standards will badly misinform ELA teachers on a number of disciplinary matters. Among the most serious are the following:

- A crucial Foundational Skills standard expecting students to read unfamiliar multisyllabic words "in context and out of context" is in grades 4 and 5 (3a) and needs to be in grades 1,2 , and 3 as well to ensure mastery of decoding skills. Its placement at only grades 4 and 5 misinforms all elementary reading teachers because it implies that decoding skills should be practiced in the primary grades only in context. This implication has the potential to set back all of beginning reading instruction despite all the fine standards in the Foundational Skills section.
- Informational reading standards need to distinguish clearly the various modes of organizing an expository text (e.g., order of time, cause and effect) from its structural elements (e.g., purpose, introduction, body, conclusion).
- The sub-strand on "argument" needs to distinguish argument from expression of opinion in the elementary grades (which is not a type of writing) and academic argument from advocacy or persuasive writing, and to clarify the key concepts at the root of persuasive writing: purpose and audience.
- The grammar standards need terminology that ELA teachers are apt to understand and more appropriate grade-level placement.
- Vocabulary standard 5c, grades 6-8: "Distinguish among the connotations (associations) of words with similar denotations (definitions)" needs to be straightened out and given appropriate examples. Teachers need to know that a word may have a denotation and a connotation (e.g.,
the head on a body and the head of the class) and that this distinction does not encompass groups of semantically related words but with different shades of meaning.

3. States should develop incentives to encourage high school sophomores or juniors deemed "college-ready" to complete grades 11 and 12 and take advanced mathematics and science courses that prepare them for authentic collegelevel coursework rather than leave high school to enroll in an open admissions institution. We need more, not fewer, high school students who can become STEM majors after they matriculate in a selective college. If they are to be prepared in high school, they need to take advanced mathematics courses in grades 11 and 12. And there need to be incentives for students to take advanced mathematics courses in grades 11 and 12. Pushing them early into a community college is unlikely to lead them to a STEM career. A 2009 report from the Massachusetts Department of Higher Education urging better preparation for college and STEM careers noted declining student interest: "Despite the increased demand for STEM careers in Massachusetts, student interest in STEM remains flat. Between 2002 and 2008 the percent of students enrolling in STEM majors remained relatively constant with an increase from 23.2 percent to 23.5 percent at the university level, a reduction in state college rates from 10.3 percent to 9.6 percent and a reduction in community college rates from 10.6 percent to 10.2 percent."

Since, according to NCEE, the most able high school students are to be encouraged to stay on in high school for Advanced Placement or other advanced courses, ${ }^{36}$ it is reasonable to assume that the students who will be encouraged to enroll in an open admissions post-secondary institution immediately (if they pass tests reflecting Common Core's standards) will come from the broad middle of the state's high school age population (i.e., students who normally complete high school graduation requirements and graduate, in contrast to those in the bottom third who drop out from
grades 9-12). These are the students who should be encouraged to complete grades 11 and 12 and take more advanced mathematics and science courses (if they are indeed college-ready after grade 10) to become part of the skilled workforce this nation needs. We might also motivate them to complete grades 11 and 12 by giving them the option of a technical/career pathway for grades 9 to 12 or in grades 11 and 12 as an alternative to a straight academic pathway

Regardless of the differences in the academic level of the standards that will undergird future high school tests, we need to ask if it is wise to encourage students in the academic middle who have been deemed "college ready" to enroll in a public college (at their own expense) before they have completed their last two years of high school (at public expense) and to bypass high school graduation requirements. If they pass mathematics tests that are not much more difficult than (or no more difficult than) the grade 10 mathematics MCAS, they will matriculate in a post-secondary institution with less mathematics knowledge than they would have had if they had first completed high school graduation requirements.
4. Congress should require the National Assessment Governing Board to conduct a High School Transcript Survey (HSTS) every two years to monitor high school mathematics course-taking. An important provision in the 2001 Elementary and Secondary Education Act required all states to participate every two years in NAEP's tests of reading and mathematics so that a common yardstick could be used to compare state test results. If the next re-authorization of ESEA requires states to use "college and career readiness standards" for accountability-oriented tests, Common Core's academically low college and career readiness standards may heavily alter current high school mathematics course-taking patterns. NAEP itself cannot measure changes in high school course-taking patterns. The best tool we have at present is its HSTS, which NAEP has used irregularly to monitor course-taking since 1987. The most recent HSTS was in 2009, and
the next one is scheduled for 2015 . This sixyear hiatus will be the longest period without a survey since these surveys began, and we need data on mathematics (and science) coursetaking in particular much sooner than 2015 if large numbers of high school students, deemed college-ready, begin to leave high school after grade 10 or 11 to enroll in an open admissions post-secondary institution instead of completing advanced mathematics and science coursework in their last year or two of high school before going on to college. Common Core's goal of closing the achievement gap by increasing the number of under-prepared students in college degree programs is likely to succeed chiefly in decreasing the number of high school students in advanced courses, and Congress must fund at least one method for detecting the effects of tests based on Common Core's standards earlier rather than later.

## Endnotes

1. Ze'ev Wurman and Sandra Stotsky. (February 2010). Why Race to the Middle? White Paper No. 52. Boston: Pioneer Institute. http://www. pioneerinstitute.org/pdf/100223 why race to the middle.pdf; R. James Milgram and Sandra Stotsky. (March 2010). Fair to Middling: A National Standards Progress Report, White Paper No. 56. Boston: Pioneer Institute. http:// www.pioneerinstitute.org/pdf/100402 fair to middling.pdf; Sandra Stotsky and Ze'ev Wurman. (May 2010). The Emperor's New Clothes: National Assessments Based on Weak "College and Career Readiness Standards," White Paper No. 61. Boston: Pioneer Institute.
2. In detailed comparison of state standards in 2005, California's mathematics standards were rated as the best and its ELA standards as second best, while the Bay State's ELA standards were rated as the best and its mathematics standards as third best. See Klein, D., Braams, B.J., Parker, T., Quirk, W., Schmid, W., \& Wilson, W.S. The State of State Math Standards 2005. Washington, DC: Thomas B. Fordham Institute, 2005. http://www.edexcellence.net/detail/news. cfm?news id=338; and Stotsky, S. The State of State English Standards. 2005. Washington, DC: Thomas B. Fordham Institute, 2005. http://www. edexcellence.net/doc/FullReport[01-03-05].pdf
3. USED's solicitation for assessment consortia seeks an equivalent to states' current high school tests (used for accountability under No Child Left Behind) and requires a commitment by the states' post-secondary institutions to use this equivalent as signifying readiness for creditbearing freshman coursework. http://www2. ed.gov/programs/racetothetop-assessment/ comprehensive-assessment-systems-app.doc
4. Brief Prospectus for a State Consortium on Board Examination Systems. From the National Center on Education and the Economy. http://www.vpaonline.org/vpa/lib/vpa/Brief Prospectus 10-30-09 V4_.pdf
5. http://eap2009.ets.org/ViewReport.asp. See also Howell, J., Kurlaender, M., \& Grodsky, E. Postsecondary Preparation and Remediation: Examining the Effect of the Early Assessment Program at California State University, November 2009. http://www.csus.edu/indiv/h/ howellj/papers/JPAMRR_Nov2009.pdf
6. As reported in Why Race to the Middle? (Wurman \& Stotsky), "In 1999, the first time the state gave an Algebra I assessment based on its 1997 academic standards, $16 \%$ of grade 8 students had taken Algebra I. By 2002, the first time the scores were calibrated to allow future comparison, $29 \%$ of grade 8 students had taken Algebra I, with $39 \%$ scoring proficient and advanced. By $2009,60 \%$ had, with $48 \%$ scoring proficient and advanced... In summary, in 2002 only 52,000 California students successfully completed Algebra 1 by grade 8. In 2009 over 139,000 students did so, almost 90,000 additional students" (p. 13).
7. http://www.doe.virginia.gov/news/news releases/2010/jun24.shtml
8. Massachusetts led all states in grades 4 and 8 in reading and mathematics on the 2005,2007 , and 2009 administrations of the National Assessment of Educational Progress (NAEP). Moreover, in a comparison of the scores of the state's lowincome students with the scores of low-income students in the other states on NAEP's 2007 state tests, low-income students in Massachusetts were tied for first place in mathematics in grades 4 and 8 and in reading in grade 4 . In grade 8 in reading, they were tied for second place. In the 2007 Trends in International Mathematics and Science Study, Massachusetts grade 4 students ranked second worldwide in science achievement and tied for third in mathematics, while its grade 8 students tied for first in science and ranked sixth in mathematics (Massachusetts had participated as an independent country).
9. Massachusetts Board of Higher Education and Massachusetts Department of Education. Massachusetts School-to-College Report: High

School Class of 2005. February 2008. This report can be found at http://www.doe.mass.edu/ research/reports/research.html and http://www. mass.edu/reports.
10. The entry-level, credit-bearing mathematics course in the Bay State's public universities and four-year colleges is called College Algebra. While College Algebra in the state's universities and four-year colleges may have more difficult content than an algebra II course, the course called College Algebra taken by students in the state's community colleges to fulfill their college mathematics requirements is viewed as equivalent to Algebra II.
11.http://www.doe.mass.edu/hsreform/masscore/ summary.doc
12.http://www.doe.mass.edu/hsreform/masscore/ intro.ppt
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14. Clifford Adelman, Answers in the Tool Box: Academic Intensity, Attendance Patterns, and Bachelor's Degree Attainment. Washington, D.C.: U.S. Department of Education, 1999. http://www. ed.gov/pubs/Toolbox/toolbox/html
15. The 2010 review form showing the rubrics for the rating scale, is in Appendix A.
16. From grades 6-8, students are to "distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., stingy, scrimping, economical, unwasteful, thrifty)." The examples given are in fact examples of synonyms, which is clear in 9-12 where the objective asks students to "analyze nuances in the meaning of words with similar denotations."

According to McDougal Littell's glossary in The Language of Literature, "Connotation is the emotional response evoked by a word, in contrast to its denotation, which is its literal meaning ..." In Handbook of Literary Terms: Literature, Language, Theory (X.J. Kennedy, D. Gioia, \& M. Bauerlein, Pearson, 2005), connotation is defined as an "association or additional meaning that a word, image, or phrase may carry, beyond its literal reference or dictionary definition. ...A rose in literature is not only the literal flower; it also carries associations attached to it, both from the many poems treating roses as metaphors for love time, etc., and from historical events... Whereas denotative meanings signify a thing with precision..., connotative meanings belong to feelings, attitudes, valuations, and biases." Common Core's standard misleads teachers.
17. Foundations for Success: The Final Report of the National Mathematics Advisory Panel. U.S. Department of Education: Washington, D.C., 2008. http://www2.ed.gov/about/bdscomm/list/ mathpanel/report/final-report.pdf
18. Hong Kong starts teaching estimation of results in grade 1 , and Korea in grade 2.
19. Also observed by others, e.g., R. James Milgram, Review of Final Draft Core Standards, testimony submitted to the California Academic Content Standards Commission, July 7, 2010, p. 2 (attached to this White Paper as Appendix B).
20. The exact point at which Common Core expects full fluency of arithmetic with integers is unclear. Grade 7 develops manipulation of positive and negative rational numbers but is unclear on expectations of fluency. Since there are no standards describing fluency with rational numbers beyond grade 7, we assume that fluency is achieved at that grade.
21. Jeremy Kilpatrick, Jane Swafford, and Bradford Findell, eds., Adding It Up, National Research Council: Washington, D.C., 2001, p. 233 ff; Curriculum Focal Points, NCTM, Reston, VA, 2006, grades 4 through 7, pp. 16-19.
22. Known as the Fundamental Theorem of Arithmetic
23. R. James Milgram, Review of Final Draft Core Standards, testimony submitted to the California Academic Content Standards Commission, July 7, 2010, pp. 4-5. Also attached as Appendix B to this report.
24. Foundations for Success: The Final Report of the National Mathematics Advisory Panel. U.S. Department of Education: Washington, D.C., 2008. http://www2.ed.gov/about/bdscomm/list/ mathpanel/report/final-report.pdf
25. Foundations for Success: The National Mathematics Advisory Panel, Reports of the Task Groups and Subcommittees. U.S. Department of Education: Washington, D.C., 2008. Chapter 3, p. 3-47 http://www2.ed.gov/about/bdscomm/list/ mathpanel/report/conceptual-knowledge.pdf
26. Milgram writes "[Common Core's] standards illustrate many serious flaws... Among these difficulties are that a large number of the arithmetic and operations, as well as the place value, standards are one, two, or even more years behind the corresponding standards for many if not all the high achieving countries" (R. James Milgram, July 2010 testimony, p. 3). Jonathan Goodman, another mathematician, writes: "The proposed Common Core standards are similar in earlier grades but have significantly lower expectations with respect to algebra and geometry than the published standards of other countries I examined." (Jonathan Goodman, A comparison of proposed US Common Core math standards to standards of selected Asian countries. http:// www.educationnews.org/ed_reports/94979.html)
27. This combined approach should not be confused with so-called "integrated mathematics" approaches in the U.S. that attempt to teach units of study combining algebra and geometry content. High achieving countries alternate geometry and algebra units lasting a few weeks or months, with each unit taught independently and brought to closure.
28. See R. James Milgram's testimony, p. 8.
29. The intended scope of the content by course can be seen in a diagram in the March 2010 draft showing the content partitioned into courses.
30. To understand more clearly what California felt it needed to add to Common Core's standards in order to meet current California high school course expectations, see the strengthened standards at http://www.scoe.net/castandards/ agenda/2010/20100722_ccs recommendations. pdf on p .127 ff .
31. The California Commission for Academic Content Standards recommended adding to Common Core's mathematics standards all of California's Calculus and Advanced Statistics standards, and supplemented the courses that students take before these two courses with numerous standards in geometry, algebra, and trigonometry.
32. http://www.doe.mass.edu/boe/docs/0710/ item1 mathpanel.doc
33. Buried at the end of the final version of its mathematics standards document is an admission of the rushed and incomplete effort it represents (p. 84). Common Core promises that "sample high school pathways for mathematics" will be made available shortly after the release of the final document on June 2. However, such suggestions, whenever they come, will have no binding power. Common Core's writing committees have been disbanded and most states will have already adopted the standards.
34. http://www.scoe.net/castandards/ agenda/2010/20100722_ccs recommendations. pdf p. 114-120.
35. These holes might be responsible in part for the difficulty that Common Core had in creating complete high school courses from the standards.
36. Brief Prospectus for a State Consortium on Board Examination Systems. From the National Center on Education and the Economy. http://www.vpaonline.org/vpa/lib/vpa/Brief Prospectus 10-30-09 V4 .pdf

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## About Pioneer:

Pioneer Institute is an independent, nonpartisan, privately funded research organization that seeks to change the intellectual climate in the Commonwealth by supporting scholarship that challenges the "conventional wisdom" on Massachusetts public policy issues.

## Appendix A: The 2010 Review <br> Form for English Language Arts Standards

## A. Reading Pedagogy and Independent Reading

1. The document expects explicit and systematic instruction in decoding skills in the primary grades as well as use of meaningful reading materials and an emphasis on comprehension.
0 Phonics or decoding skills are not mentioned at all.
1 Phonics or decoding skills are mentioned only in the context of other strategies so that it is unlikely they are addressed independently or systematically.
3 Phonics or decoding skills are given a separate bullet or statement but there is nothing on explicit and systematic teaching and independence from contextual approaches.

4 Explicit and systematic instruction in decoding skills, both independent of context and in context, is spelled out as such.
2. The standards make clear that interpretations of written texts should be supported by logical reasoning, accurate facts, and adequate evidence.

0 The standards imply that all points of view or interpretations are equally valid regardless of the logic, accuracy, and adequacy of supporting evidence.

1 The standards imply that all literary texts are susceptible of many equally valid interpretations.
3 The standards indicate that interpretations of texts must be in part on what is in the texts.
4 The standards indicate that interpretations of any text must be consistent with what the author wrote.
3. The document expects students to read independently on a daily basis through the
grades, and provides guidance about quality and difficulty.

0 Independent reading isn't mentioned at all.
1 Regular independent reading is recommended, but not quality, quantity, or difficulty.
3 Quality, quantity, or difficulty of independent reading is indicated in some way (e.g., by a list of recommended books or by a recommended number of words or books per grade).

4 Quality, quantity, and difficulty are indicated in some way.

## B. Value Accorded Literary Study

1. The standards enable English teachers to stress literary study more than informational reading, especially at the secondary level.
0 Literary standards are not distinguished from non-literary standards.
1 Literary reading is distinguished from non-literary reading but is stressed less than non-literary reading at higher grade levels.
3 Literary reading is stressed about equally with non-literary reading at higher grade levels.
4 The emphasis on literary reading is greater than the stress on non-literary reading at higher grade levels.
2. The document indicates that what students are assigned to read should be chosen on the basis of literary quality, cultural and historical significance, orpotentialto promoteunderstanding of more complex ideas and language.
0 The standards expect the literary and nonliterary texts that students are assigned to read to address their daily lives and contemporary social issues.

1 The standards expect some of the literary and non-literary texts that students are assigned to address their daily lives and contemporary social issues.

3 The standards expect much or most of the literary and non-literary texts that students are assigned
to address their daily lives and contemporary issues.

4 The standards do not expect the literary and non-literary texts that students are assigned to read to address their daily lives and contemporary social issues.
3. The standards promote study of American literature.
0 American literature is not mentioned as such in any way.
1 American literature is mentioned, but no more than that.
3 American literature is mentioned in an inclusive way.
4 American literature is described in an inclusive way with illustrative works or authors.

## C. Organization and Disciplinary Coverage of the Standards

1. They are grouped in categories and subcategories reflecting coherent bodies of scholarship or research in reading and the English language arts.
0 They are mostly grouped in unique or incoherent categories or subcategories (e.g., categories reflect pedagogical strategies).
1 Some categories or subcategories reflect coherent bodies of scholarship or research.
3 Most but not all categories and subcategories reflect coherent bodies of scholarship or research.

4 All categories and subcategories reflect coherent bodies of scholarship or research.
2. The standards clearly address listening and speaking. They include use of various discussion purposes and roles, how to participate in discussion, desirable qualities informal speaking, and use of established as well as peer-generated or personal criteria for evaluating formal and informal speech.

0 Standards for listening and speaking are not included.

1 Some of the above areas for coverage are addressed adequately.

3 Most but not all of the above areas are addressed adequately.

4 All of the above areas are adequately covered.
3. The standards clearly address reading to understand and use information through the grades. They include progressive development of reading skills, knowledge and use of a variety of textual features, genres, and reading strategies for academic, occupational, and civic purposes.
0 Standards for informational (including technical, persuasive, and procedural) reading are not distinguished as such.
1 Some of the above areas for coverage are addressed adequately.
3 Most of the above areas for coverage are addressed adequately.
4 All of the above areas are adequately covered.
4. The standards clearly seek to develop strong vocabulary knowledge and dictionary skills.

0 Vocabulary standards are not in a distinct strand for instruction.

1 Vocabulary standards emphasize use of context but do not offer specifics on how to use context

3 Vocabulary standards highlight specific figures of speech and rhetorical devices but are limited in the categories of words they highlight.
4 Vocabulary standards teach dictionary skills, use of glossaries for the meaning of disciplinespecific terms, ways to use context, and all useful categories of phrases, words, or word parts (e.g., foreign words, idioms, proverbs).
5. The standards clearly address the reading, interpretation, and critical evaluation of literature. They include knowledge of diverse literary elements and genres, different kinds
of literary responses, and use of a variety of interpretive and critical lenses. They also specify those key authors, works, and literary traditions in American literature and in the literary and civic heritage of English-speaking people that all students should study because of their literary quality and cultural significance.
0 Standards for literary study are not distinguished as such.

1 Some of the above areas for coverage are addressed adequately.
3 Most of the above areas for coverage are addressed adequately.
4 All of the above areas are adequately covered.
6. The standards clearly address writing for communication and expression. They include use of writing processed, established as well as peergenerated or personal evaluation criteria, and various rhetorical elements, strategies, genres, and modes of organization.
0 Standards for writing for communication and expression are not distinguished as such.

1 Some of the above areas for coverage are addressed adequately.
3 Most of the above areas for coverage are addressed adequately.
4 All of the above areas are adequately covered.
7. The standards clearly address oral and written language conventions. They include standard English conventions for sentence structure, spelling, usage, penmanship, capitalization, and punctuation.
0 Standards for oral and written language conventions are not distinguished as such.
1 Some of the above areas for coverage are addressed adequately.
3 Most of the above areas for coverage are addressed adequately.
4 All of the above areas are adequately covered.
8. The standards clearly address the nature, dynamics, and history of the English language. They include the nature of its vocabulary, its structure (grammar), the evolution of its oral and written forms, and the distinction between the variability of its oral forms and the relative permanence of its written form today.
0 Standards for this area are not distinguished as such.

1 Some of the above areas for coverage are addressed adequately.
3 Most of the above areas for coverage are addressed adequately.
4 All of the above areas are adequately covered.
9. The standards clearly address research processes, including developing questions and locating, understanding, evaluating, synthesizing, and using various sources of information for reading, writing, and speaking assignments. These sources include dictionaries, thesauruses, other reference materials, observations of empirical phenomena, interviews with informants, and computer databases.
0 Standards for the research processes are not distinguished as such.
1 Some of the above areas for coverage are addressed adequately.
3 Most of the above areas for coverage are addressed adequately.
4 All of the above areas are adequately covered.

## D. Quality of the Standards

1. They are clear, specific, and measurable (i.e., they can lead to observable, comparable results across students and schools).

0 They are vague, filled with jargon, and/or expressed in ways that are not measurable (e.g., use unmeasurable verbs like "explore," "investigate," "inquire," or ask for personal experience).

1 To some extent, clear, specific, teachable, measurable, and reliably rated.
3 For the most part, clear, jargon-free, teachable, and measurable, and reliably rated.
4 Overall, they are clear, jargon-free, teachable, measurable, and reliably rated.
2. They are of increasing intellectual difficulty at each higher educational level and cover all important aspects of learning in the area they address.
0 For the most part, they show little change in difficulty over the grades, or are frequently repeated for many grades at a time.
1 Increases in difficulty are reflected to some extent by substantive changes in the wording of a standard or by new standards pointing to more difficult content.
3 Most of the standards show meaningful increases in difficulty over the grades and address the important aspects of learning in the area.
4 Overall, the standards show educationally appropriate and meaningful increases in difficulty over the grades and cover all important aspects of learning in the area.
3. They index or illustrate growth through the grades for reading by referring to specific reading levels or to titles of specific literary or academic works as examples of a reading level.
0 The reading and/or literature standards contain no clue as to reading level other than something like "using texts at the appropriate grade level."
1 The reading and/or literature standards are occasionally accompanied by examples of specific texts or authors.
3 The reading and/or literature standards are frequently accompanied by examples of specific texts or authors.
4 The reading and/or literature standards are almost always accompanied by examples of specific texts and/or authors, or reading lists
divided into educational levels.
4. They illustrate growth expected through the grades for writing with reference to examples and rating criteria, in the standards document or in other documents.
0 The document provides no criteria or samples for the quality of writing expected at assessed grades.
1 The document provides criteria or examples for the quality of writing expected at most but not all assessed grades through high school.
3 The document provides criteria or examples for the quality of writing expected at all assessed grades through high school.
4 The document provides examples and criteria for the quality of writing expected at all assessed grades, including high school.
5. Their overall contents are sufficiently specific, comprehensive, and demanding to lead to a common core of high academic expectations for all students.
0 No. They cannot lead to a common core of high academic expectations.
1 To some extent only.
3 For the most part.
4 Yes.

[^4]
# Review of Final Draft Core Standards 

R. James Milgram

What follows are my comments on the final draft of the CCSSI Core Mathematics Standards. There are a number of standards including, but not limited to 1-OA(6), 2$\mathrm{OA}(2), 2-\mathrm{NBT}(5), 3-\mathrm{OA}(7), 3-\mathrm{NBT}(2), 4-\mathrm{OA}(4), 4-\mathrm{OA}(6), 4-\mathrm{NF}(1), 4-\mathrm{NF}(2), 5-\mathrm{OA}(3)$, $8-\mathrm{G}(2), 8-\mathrm{G}(4), \mathrm{F}-\mathrm{LQE}(5)$, G-SRT(4) that are completely unique to this document, and most of them seem problematic to me. I have repeatedly asked for references justifying the insertions of these or similar standards in previous drafts, but references have not been provided. Consequently, to my knowledge, there is no real research base for including any of these standards in the document.

## Basic Arithmetic and Arithmetic Operations.

Here are $1-\mathrm{OA}(6), 2-\mathrm{OA}(2), 3-\mathrm{OA}(7), 2-\mathrm{NBT}(5), 3-\mathrm{NBT}(2), 4-\mathrm{OA}(4)$, and $4-\mathrm{OA}(6)$ :
1.OA(6) Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8+6=8+2$ $+4=10+4=14$ ); decomposing a number leading to a ten (e.g., $13 \quad 4=13 \quad 31$ $=101=9$ ); using the relationship between addition and subtraction (e.g., knowing that $8+4=12$, one knows $128=4$ ); and creating equivalent but easier or known sums (e.g., adding $6+7$ by creating the known equivalent $6+6+1=12+1=13$ ).
2.OA(2) Fluently add and subtract within 20 using mental strategies.

2 By end of Grade 2, know from memory all sums of two one-digit numbers.
2-NBT(5) Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

3-NBT(2) Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3-OA(7) Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $85=40$, one knows $40 \div 5=$ 8 ) or properties of operations. By the end of Grade 3 , know from memory all products of two one-digit numbers.

4-OA(4) . Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 ?=48,5=? 3,66=?$.

4 -OA(6) . Understand division as an unknown-factor problem. For example, find $32 \div 8$ by
finding the number that makes 32 when multiplied by 8 .

## Note that

- most of these standards have some sort of fluency requirement for operations in a range, but no requirement that the algorithm being used is either general or generalizable. Also,
- note the extremely excessive pedagogical constraints in 1-OA(6), 3-OA(7).
- Note that $4-\mathrm{OA}(6)$ is actually a definition, and part of a definition that is given at least one year earlier in virtually all the high achieving countries at that.

Specifically, subtraction is defined in the following way: $a-b$ is that number $c$, if it exists, so that $b+c=a$, while division is defined by $a \div b$ is that number, $c$, if it exists, so that $b \times c=a$.

With these understandings, the students in the high achieving countries only have to learn and master two operations, addition and multiplication, since the other two come along for free. Moreover, this is a key piece of the underpinnings for their success. But we are, instead, given $4-\mathrm{OA}(6)$ which is neither fish nor fowl.

As regards fluency, I note that ultimately with
4-NBT(4) Fluently add and subtract multi-digit whole numbers using the standard algorithm.

5-NBT(5) Fluently multiply multi-digit whole numbers using the standard algorithm.
6-NBT(2) Fluently divide multi-digit numbers using the standard algorithm.
expectations are that students will fluently operate with reasonable variants of the standard algorithms. But what will be the effects of the previous fluency requirements, except long-term confusion about key details of what is to be expected? So we can well imagine average and weaker students using some weird mnemonics to handle operations in certain ranges, and trying to combine this with a kind of dim understanding of how the standard algorithms work.

To further add to the confusion surrounding these core standards, note the following entirely reasonable standards that can only be regarded as competing with the "fluency" standards within the document.

2-NBT(6) Add up to four two-digit numbers using strategies based on place value and properties of operations.

2-NBT(7) Add and subtract within 1000, using concrete models or drawings and strategies based
on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2-NBT(9) Explain why addition and subtraction strategies work, using place value and the properties of operations This is almost certainly too advanced for second grade, but indicates a viable direction for student exploration in this and later grades.

3-NBT(3) Multiply one-digit whole numbers by multiples of 10 in the range 1090 (e.g., $9 \times 80$, $5 \times 60$ ) using strategies based on place value and properties of operations.

4-NBT(5) Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4-NBT(6) Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5-NBT(6) Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

The seven standards above would have been exemplary if they had not ocurred after the "fluency" standards for unconstrained algorithms that I had objected to at the beginning of this discussion. Within the document itself, there seems to be a minor war going on, and this is not something we should hand over to our teachers.

The above standards illustrate many serious flaws in the Core Standards. Also among these difficulties are that a large number of the arithmetic and operations, as well as the place value standards are one, two or even more years behind the corresponding standards for many if not all the high achieving countries. Consequently, I was not able to certify that the Core Mathematics Standards are benchmarked at the same level as the standards of the high achieving countries in mathematics.

## FRACTIONS

Just as there are serious concerns with the coherence of the Core Standards for basic arithmetic and place-value, there are also concerns with the coherence of the Core Standards for fractions, though here the difficulties are somewhat more subtle. Fractions first appear in grade 3 in Core Standards which is somewhat late by international expectations, but not too out of line.

3-NF (1) Understand a fraction $1 / b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a / b$ as the quantity formed by a parts of size $1 / b$.

3-NF(2) Understand a fraction as a number on the number line; represent fractions on a number line diagram.
a. Represent a fraction $1 / b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1 / b$ and that the endpoint of the part based at 0 locates the number $1 / b$ on the number line.
b. Represent a fraction $a / b$ on a number line diagram by marking off $a$ lengths $1 / b$ from 0 . Recognize that the resulting interval has size $a / b$ and that its endpoint locates the number $a / b$ on the number line.

3-NF(3) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., $1 / 2=2 / 4,4 / 6=2 / 3$ ). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3=3 / 1$; recognize that $6 / 1$ $=6$; locate $4 / 4$ and 1 at the same point of a number line diagram.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

Remark. This "visual fraction model" represents all that is wrong in our standard approach to fractions - an approach that has seldom worked. From the glossary, we have

Visual fraction model. A tape diagram, number line diagram, or area model.
In short, what is done is to use three separate and basically unconnected models for fractions to decide if statements are true, false, or ambiguous. In particular, referring back to $3-\mathrm{NF}(3)$, we have the separate notions of position on the number line and size. These are initially very different concepts when matched to student experience with numbers. Size refers to counting, but when dealing with fractions, counting is not appropriate except in the "partitive" model, which is abandonded very early in the development of this subject in the high achieving countries.

Indeed, what is done in the high achieving countries is to refer fractions entirely back to the number-line as soon as this becomes feasible - usually sometime in second grade or at the beginning of grade 3 - and not refer to size except in-so-far as a number on the number line to the right of another number is said to be larger.

Another point where the handling of fractions is problematic is in fourth grade:
4-FN(1) Explain why a fraction $a / b$ is equivalent to a fraction $(n \times a) /(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

There are many ways to handle this, but visual fraction models is pretty much the worst. One thing that can be done is to observe that the point on the number line associated to $(n \times a) /(n \times b)$ is exactly the same as the point associated to $a / b$ provided $b \neq 0$. So "equivalent fraction" can be taken to mean "fraction representation by the same point on the number line," and, again, in the high achieving countries, this is the approach taken:

For information, here is the teaching sequence in grades 2-4 for fractions in Singapore:
Teaching sequence, Singapore, grades 2-4 The initial presentation of fractions in the Singapore programs occurs in the second half of second grade, and is developed using an area model where care is taken to be sure that the regions that decompose a geometric
figure are have the same area:
4. What froction of each shope is coloured?
$x$ n
(a)

(b)

(d)

明
AI


6

$\frac{1}{4}$ ons Th mose 1 -atole.
(c)


In the second part of the grade three text fractions continue to be developed using an area model, but the level of sophistication as increased significantly:
9.


The fractions $\frac{3}{9}, \frac{5}{9}$ and $\frac{7}{9}$ have a common denominator.
is the smallest fraction.
$\square$ is the greatest fraction.
and equaivalent fractions are introduced

$\frac{1}{2}, \frac{2}{4}$ and $\frac{4}{8}$ are equivalent fractions.


In the fourth grade the area model is moved systematically towards seeing fractions on the number line as the basic operations of addition and subtraction of fractions are developed:
7. Add $\frac{1}{5}$ and $\frac{3}{10}$.

3. Change the improper fractions to mixed numbers.


Remark: In the Russian texts translated by UCSMP the sequencing is very similar except that representing fractions on the number line is already present in grade 3.

The next problem is with the standard
4-FN(2) Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>,=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

The first part of this standard is exemplary, but it is completely distorted by what follows. What does in mean to compare to "a benchmark fraction?" And this is only made worse by the requirement that students "recognize that comparisons are valid only when two fractions refer to the same whole." This is an entirely unappetizing admixture of apples and spoiled oranges.

## Geometry

The approach to geometry in Core Standards is very unusual, focusing in eighth grade and beyond on using the Euclidian and extended Euclidean groups to define congruence and similarity as well as develop their key properties. Mathematically, this approach is rigorous, but it would generally be regarded as something that would be done in a college level geometry course for math majors. The exposition at the high school level seems plausible, and may well work. However, to my knowledge, there is no solid research that justifies this approach at the K-12 level currently.

It is also worth noting that a similar approach was taken in Russia about 30 years back, but was quickly rejected. It wasn't that the teachers were not capable of teaching, though this may well be a problem for most middle school and many high school math teachers here. The problem was that it was way too non-standard, and basic geometric facts and theorems had to be handled in entirely new, untested, and ultimately unsuccessful ways.

Here are some details on the issues that arise in geometry.
3-MD(5) Recognize area as an attribute of plane figures and understand concepts of area measurement.
a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units.

Of course, the basic issue is that most figures in the plane are not decomposed into $n$ unit squares without gaps or overlaps. For example, what of the triangle? 3-MD(5) is a good beginning for the discussion of area, but it is not more than this.

In fourth grade we have
4-MD(1) Know relative sizes of measurement units within one system of units including km, $\mathrm{m}, \mathrm{cm} ; \mathrm{kg}, \mathrm{g} ; \mathrm{lb}$, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a twocolumn table. For example, know that 1 ft is 12 times as long as 1 in . Express the length of a 4 ft snake as 48 in . Generate a conversion table for feet and inches listing the number pairs $(1,12),(2,24),(3,36), \ldots$

This is the summative standard for a whole sequence of standards that start in the earliest grades but continue through grade 5 or even grade 6. It is far too complex to be listed only in grade 4 . But that is exactly what is done in Core Standards. It is as though the authors had a master-list of topics and felt free to sprinkle them wherever there might have been room.

In grade 5 the analogue of $3-\mathrm{MD}(5)$ is presented:
5-MD(3) Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
b. A solid figure which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.

Partly, I feel that this standard is occuring too early. It takes some time and effort for students to appreciate the complexity of visualizing solid figures through plane sections or possibly nets. Partly, as before, this standard is avoiding the real issues, namely, determining the volumes of figures that can not be decomposed into $n$ cubes without gaps or overlaps, such as triangular prisms or rectangular cones. When we look at this pair of issues together, we can begin to see why I feel so uncomfortable with these standards.

At the same time, look at the geometry standards $5-\mathrm{G}(3)$ and $5-\mathrm{G}(4)$.
5-G(3) Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

5-G(4) Classify two-dimensional figures in a hierarchy based on properties.
Except for parsing the convoluted language of the first, both of these standards are at an astoundingly trivial level for fifth grade. By this time students should be comfortable with the area formula for a triangle, and should be constructing compound two and three dimensional figures as well as determining a number of their properties.

In eighth grade the experimental approach to geometry that I mentioned earlier manifests for the first time. First there is a very superficial development of the properties of some Euclidian transformations in the plane:

8-G(1) Verify experimentally the properties of rotations, reflections, and translations:
a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Then, based entirely on the relatively weak standard above we are directly given one of the most subtle definitions of congruence we could possibly find.

8-G(2) Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

It is not the first piece of $8-G(2)$ that disturbs me - though there are a number of key steps that are hidden within it - but the second: given two congruent figures, describe a sequence that exhibits the congruence between them. (By "a sequence" I am presuming the writers meant "a sequence of rotations, reflections and translations.") What is being hidden here is the result that is deep even at the level of a university course in geometry: given two congruent figures, then there exists a Euclidean transformation that takes the first to the second, and a Euclidean transformation that takes the second to the first.

It is at the point above, and even more so with the corresponding similarity standard
8-G(4) Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them
where I feel that we are dealing with an experiment on a national scale. There are even more difficulties with the statement "given two similar two dimensional figures, describe a sequence that exhibits the similarity between them" than was the case with the corresponding statement in $8-\mathrm{G}(2)$.

Before we dare to challenge teachers and students with standards like these, we absolutely have to test the approach in more limited environments, and I find it highly disturbing that H.-H. Wu, the main author of the greometry standards in Core Standards, feels able to make the following statement in a recent article:

The mathematical coherence of CCMS also lies at the heart of the discussion of high school geometry. Briefly, the better standards, such as Californias, insist on teaching proofs. This is a good thing, but it does place an unreasonable burden on a high school course on geometry as the only place where any kind of proof can be found in school mathematics. As a result, some of these courses begin with formal proofs based on axioms from the beginning, with no motivation. There is another kind of reaction, however. Giving up entirely on proofs as unlearnable, some courses treat plane geometry as a sequence of hand-on activities that do not mention proofs. In addition, both kinds of courses are disconnected from the teaching of rigid motions (translations, rotations, and reflections) in middle school. What CCMS does is to add the teaching of dilations to rigid motions in grade 8 using hands-on activities, and on this foundation, develops high school geometry by proving all the traditional theorems. For the first time, the school geometry curriculum provides a framework in which all the apparently unrelated pieces of information now begin to form a coherent whole. It holds the promise that learning geometry in K-12 can finally become a reality.

Over the last 12 years Wu and I have collaborated on the California Framework, a number of other states standards, and on a number of nationally influential documents. Normally, Wu is very careful about distinguishing between what one hopes is true and what one knows will work, but in this instance I feel he has allowed his innate hope to overwhelm caution.

## Eighth Grade Algebra

Another issue with the Core Math Standards is that there are no provisions for eighth grade algebra. This contrasts with the California standards where the expectation is that most students will be ready for Algebra I by eighth grade.

Moreover, as the following graph shows, eighth grade Algebra I is basically working already, with almost $60 \%$ of California's students taking the course either in seventh or eighth grade.

Algebra 1 Proficiency In Elghth Grade or Earller


It is worth noting that setting standards up so that Algebra I occurs naturally by eighth grade involves a large amount of preparatory material including basic pre-algebra standards and certain key geometry standards, such as understanding that the graph of a linear equation is a straight line. So it is far from sufficient to just list key algebra topics and decree a course that covers them.

As regards the Core Standards this is an issue I, as a member of the Pathways Committee, as well as the Validation Committee, have been struggling with for months. We have been able to rough in courses that are mostly based on the High School Core Standards, which will work, but we are far from finished with this project.

## Final Remarks

There are also very real strengths in the document. Many of the discussions, among them ratio and rate in grade 6, and proportion in grade 7, are excellent. They are clear and mathematically correct presentations of material that is typically very badly done in most state standards in this country.

Overall, only the very best state mathematics standards, those of California, Massachusetts, Indiana and Minnesota are stronger than these standards. Most states would be far better off adopting the Core Math Standards than keeping their current standards. However, California, and the other states with top standards would almost certainly be better off keeping their current standards.

In the following pages I detail the comments, organized by grade level, that I had with regard to the final draft of Core Standards. Since many of my objections were not addressed in the two days before the final version was publicly released, the full list may have some use.



[^0]:    CC=Common Core 2010 CA= California 1998
    MA= Massachusetts 2001/ 2004
    DMA= Draft Massachusetts 2010

[^1]:    ${ }^{i}$ Although one of the major justifications for adopting Common Core's standards given to the media by Massachusetts Secretary of Education Paul Reville and Commissioner of Education Mitchell Chester is the claim that specialists from the Department of Elementary and Secondary Education contributed heavily to the content and shape of Common Core's standards, implying that this has made for few differences between Common

[^2]:    - By the end of Grade 5, students should be proficient with comparing fractions and

[^3]:    ${ }^{\text {ii }}$ The authors thank Jonathan Goodman for this observation.

[^4]:    Appendix B: Review of Final Draft Core Standards by R. James Milgram. Testimony to the California Academic Content Standards Commission, July 7, 2010.

